USA Mathematical Talent Search

Round 3 Grading Rubric

Year 37 — Academic Year 2025–2026

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GENERAL GUIDELINES

1. The grading rubric is designed to be simultaneously **specific** and **flexible**. For common solution methods, the rubric provides a specific allocation of points to ensure consistency across graders. Less common solution methods might not be captured closely by the rubric. For less common solution methods, consider the amount of constructive progress (including any specific intermediate results discussed in the rubric) and how far or close the student is to a complete solution when determining the score.
2. On **all** problems, the graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.
3. Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem.
4. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. If a student uses WolframAlpha, they must show their inputs and outputs. Merely citing the name of a software package is not sufficient justification.
5. A student's justification needs to be rigorous and reasonably clear in order for the solution to earn **5 points**. If there is a meaningful gap in the student's argument or a key step is unclear, deduct points accordingly.

Problem 1/3/37:

Award **5 points** for the correct configuration of numbers. No justification is required. Withhold **1 point** for each number that is incorrect.

Problem 2/3/37:

Note: Award a total score of **1 point** if the student obtains the correct answer with minimal or no explanation.

Solution 1:

1 point: Student suggests analyzing the "smallest variant" and makes a reasonable start to analyzing this game.

1 point: Student proves that if n is odd, then $a_n = b_n$.

1 point: Student proposes the formulas for b_n for both even and odd n. (Do not award any credit for just one of the formulas.)

1 point: For odd n, student finds b_n in terms of b_{n-1}.

1 point: Student completes the induction.

Solution 2:

1 point: Student finds that the probability the card labeled k is put into the scoring pile is $\frac{1}{n-k+1}$.

2 points: Student finds $p(x)$ in terms of n.

1 point: Student finds that $p_0 + p_1 + p_2 + p_3 + \cdots = p(1) = 1$ and $p_0 - p_1 + p_2 - p_3 + \cdots = p(-1) = -\frac{1}{n}$. (The student needs both results to get this point.)

1 point: Student finds that $p_0 + p_2 + \cdots = \frac{n-1}{2n}$ and arrives at the correct answer for $n = 101$.

Note: Withhold **1 point** if the student does not discuss that the probabilities of different cards appearing in the scoring pile are independent.

Problem 3/3/37:

Solution 1:

2 points: Student proves the lemma that $[1, 2a]$ is good if a has no 2s in its base 3 representation. Award **1 point** for each of Case 1 and Case 3.

1 point: Student proves the corollary that $[2a + 1, 2b]$ is good if both a and b have no 2s in their base 3 representation.

2 points: Student completes the proof. Award **1 point** of partial credit for recognizing that the corollary gives us a way to write m in terms of balanced ternary. Stating the claim (a proof is not needed) that all positive integers can be written using balanced ternary then completes the proof.

Solution 2:

1 point: Student uses a meaningful example to build an intuition, such as recognizing how we can "triple the interval" from $[3, 8]$ to $[7, 26]$ or go from $2m$ elements to $2m + 2$ elements. For the latter, the student needs to explore a non-trivial case that includes removing elements in addition to adding elements. For example, we can go from $[1, 8]$ to $[9, 18]$.

1 point: Student makes reasonable progress towards a generalization. This could include recognizing that if $[a, b]$ contains $2m$ elements, then $[3a, 3b]$ has $2(3m - 1)$, $[3a, 3b + 2]$ has $2(3m)$, and $[3a - 2, 3b + 2]$ has $2(3m + 1)$ elements.

3 points: Student proves a generalization that completes the solution with a rigorous inductive argument. Award **1 point** of partial credit for significant constructive progress and **2 points** of partial credit for a nearly complete solution.

Problem 4/3/37:

1 point: Student makes a good diagram that captures the problem setup. (This point was typically withheld if there was no diagram and a diagram would have been helpful to the grader.)

1 point: Student shows that AB is parallel to H_AH_B.

1 point: Student shows that AH_A, BH_B, and CH_C concur at the center of homothety.

1 point: Student shows that O is the orthocenter of triangle $H_AH_BH_C$.

1 point: Student completes the proof by explaining why HO also passes through the center of homothety.

Problem 5/3/37:

2 points: Student shows that if $N = 2^m \cdot 3$ for $m \geq 1$, then $A(N) = B(N)$. Award **1 point** of partial credit for significant constructive progress towards this result. An example would be a key intermediate result with proof such as Lemma 1.

3 points: Student shows that if $A(N) = B(N)$, then either N is prime or $N = 2^m \cdot 3$. Award partial credit as appropriate for meaningful intermediate results with proof, and for significant constructive progress towards the proof of a key intermediate result.

For student solutions similar to the official solution, a result such as Lemma 2 with proof would be worth **2 points**. The final **1 point** would be for using the lemmas to complete the proof.

For a less significant but still meaningful intermediate result with proof, award **1 point** of partial credit.

Note: If a student doesn't recognize that $A(N) = B(N)$ when N is prime, award a maximum score of **4 points**.

Note: Award a total score of **1 point** if the student obtains the correct answer with minimal or no explanation.