USA Mathematical Talent Search
Round 3 Grading Rubric
Year 36 — Academic Year 2024–2025
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GENERAL GUIDELINES

1. The grading rubric is designed to be simultaneously **specific** and **flexible**. For common solution methods, the rubric provides a specific allocation of points to ensure consistency across graders. Less common solution methods might not be captured closely by the rubric. For less common solution methods, consider the amount of constructive progress (including any specific intermediate results discussed in the rubric) and how far or close the student is to a complete solution when determining the score.
2. On **all** problems, the graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.
3. Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem.
4. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. If a student uses WolframAlpha, they must show their inputs and outputs. Merely citing the name of a software package is not sufficient justification.
5. A student's justification needs to be rigorous and reasonably clear in order for the solution to earn **5 points**. If there is a meaningful gap in the student's argument or a key step is unclear, deduct points accordingly.

Problem 1/3/36:

Award **5 points** for the correct configuration of entries. No justification is required. Withhold **1 point** for each entry that is incorrect (shaded instead of unshaded, or vice versa).

Problem 2/3/36:

1 point: Student computes the relative speed of the minute hand relative to the hour hand, which is $\frac{11}{2}^\circ$/min.

1 point: Student determines that it always takes $\frac{720}{11}$ minutes before the minute and hour hands have the same angle between them.

1 point: Student recognizes that in a 12-hour (720-minute) period, there are exactly 11 pairs in S whose angles have a difference of θ, and these 11 values for m are evenly spaced along the clock face.

1 point: Student recognizes that the minimal number of degrees the math beasts must turn the hands is $\frac{360^\circ}{11} - \alpha$.

1 point: Student computes the expected value of $\frac{180^\circ}{11}$.

Note: To earn the last **2 points**, it's sufficient to explain why the expected value must be $\frac{360^\circ/11}{2}$ without introducing α.

Note: Award a total score of **1 point** if the student provides the correct answer with little or no explanation.

Problem 3/3/36:

Note: Part (a) is worth **3 points** and part (b) is worth **2 points**.

1 point: Student squares the given equation and recognizes that x^2 is an even perfect square.

1 point: Student uses the fact that x^2 is an even perfect square to find a useful expression for b, such as $4\ell^2(a - \ell^2)$.

1 point: Student recognizes that if $\ell = 1$, then b is positive if $a \geq 2$.

2 points: Student correctly solves part (b). Since the problem says "for all sufficiently large a," students should specify their lower bound for a with proof, but it is not necessary for students to obtain the same bound $a \geq 8$ that we have in the official solution. Award **1 point** of partial credit for significant constructive progress towards a correct solution, or if the student's expression for b in terms of a is correct but the bound is incorrect.

Problem 4/3/36:

Note: A common student question during the round was about situations in which P is not strictly inside $\triangle ABC$. Since we are only concerned with the situations in which P satisfies the conditions in the problem, students don't need to address other scenarios.

1 point: Student draws a useful diagram that conveys the given information and that $ABCD$ is a cyclic quadrilateral. Withhold this point if the student doesn't include a (good) diagram, but a diagram would have been helpful.

1 point: Student builds on the fact that $ABCD$ is a cyclic quadrilateral to obtain some useful intermediate results, such as that $\overline{A'A} \parallel \overline{BD} \parallel \overline{C'C}$.

1 point: Student shows that $PB = PD$.

1 point: Student shows that $\overline{PB} \perp \overline{PD}$ if and only if P is the center of ω.

1 point: Student completes the proof by showing that P is the center of ω if and only if $\overline{AC} \perp \overline{BD}$.

Note: Students need to prove an "if and only if" statement. If only one direction is proven, award at most **4 points**. The amount of points awarded should depend on what is needed to complete the other direction of the proof.

Note: For the direction $\overline{PD} \perp \overline{PB} \implies \overline{AC} \perp \overline{BD}$, having $\angle DOB = \angle DPB$ is not enough to conclude that $O = P$. The student also needs to prove that $PD = PB$. This was a common pitfall of isogonal conjugate solutions.

Problem 5/3/36:

Note: Students set up their casework in different ways. The rubric below covers the official solution, but if the student does casework in a different way, award partial credit based on the difficulty of the cases the student covered correctly and those the student did not. For example, in the official solution $a = 1$ is by far the most involved case so it is worth **3 points**.

1 point: Student shows that $a = 1$ or $a = 2$.

1 point: Student shows that if $a = 2$, then the only solution is $(a, b, c) = (2, 0, 1)$.

3 points: Student performs a rigorous and complete analysis of the $a = 1$ case. Award partial credit for significant constructive progress as follows. Award **1 point** for an analysis using multiple mods that builds significantly towards the equation $10 \cdot 5^{12h} - 9 \cdot 3^{12g} = 1$ or another useful result. Award **1 point** for proving a key intermediate result, such as the Claim in the official solution. Award **1 point** for explaining how the claim tells us that $(a, b, c) = (1, 1, 2)$ is the only solution with $a = 1$ other than the easier-to-find solution $(a, b, c) = (1, 0, 0)$.

Note: Award a total score of **1 point** if the student obtains all three solutions with minimal or no explanation. If the student does not find all three solutions, award at most **4 points**.

Note: If the student creates a computer program that finds all three solutions but tests only finitely many possibilities, award a total score of **1 point**.

Note: Zsigmondy's theorem and the Lifting-The-Exponent lemma are valid approaches to this problem. Examples of these approaches have been added to the official solutions.

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Please visit **https://www.usamts.org** for details about solution submission.