USA Mathematical Talent Search
Round 1 Grading Rubric
Year 36 — Academic Year 2024–2025
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GENERAL GUIDELINES

1. The grading rubric is designed to be simultaneously **specific** and **flexible**. For common solution methods, the rubric provides a specific allocation of points to ensure consistency across graders. Less common solution methods might not be captured closely by the rubric. For less common solution methods, consider the amount of constructive progress (including any specific intermediate results discussed in the rubric) and how far or close the student is to a complete solution when determining the score.
2. On **all** problems, the graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.
3. Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem.
4. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. If a student uses WolframAlpha, they must show their inputs and outputs. Merely citing the name of a software package is not sufficient justification.
5. A student's justification needs to be rigorous and reasonably clear in order for the solution to earn **5 points**. If there is a meaningful gap in the student's argument or a key step is unclear, deduct points accordingly.

Problem 1/1/36:

Award **5 points** for the correct configuration of entries. No justification is required. Withhold **1 point** for each entry that is incorrect. An incorrect entry consists of a cell filled with an incorrect number, a cell that has a number but should be empty, or a cell that is empty but should have a number. Accordingly, if two entries are switched, withhold **2 points**. Do not withhold points if an entry given in the problem statement is filled in incorrectly.

Problem 2/1/36:

2 points: Student creates a clear diagram that shows the correct geometric relationship between the regular hexagon and the unit circle. Award **1 point** if the student describes the correct geometric configuration in words, but does not include a diagram.

3 points: Student finds the area of the regular hexagon. Award **1 point** of partial credit if the student makes significant constructive progress, such as finding the side length of the hexagon. Award an additional **1 point** for further significant constructive progress, such as finding the area of one of the six congruent equilateral triangles that make up the hexagon.

Note: If the student has an incorrect geometric relationship between the regular hexagon and the unit circle, award at most **1 point**.

1 point: Student obtains the correct answer of $\frac{\sqrt{3}}{2}$. No explanation is required to earn this point.

Problem 3/1/36:

1 point: Student obtains the correct answer of $M = 3$. No explanation is required to earn this point.

2 points: Student provides a construction in which $M = 3$. As long as it is easy to see that the construction works, award both points. If the student has an incorrect construction, but provides a detailed explanation of how they came up with their construction that contains significant constructive progress towards a correct construction, award **1 point**.

2 points: Student shows that $M = 3$ is guaranteed. Award **1 point** for significant constructive progress towards this result, such as recognizing that by the Pigeonhole Principle at least three of the numbers 1, 2, 3, 4, 5 must be in the same group.

Problem 4/1/36:

Note: Students structured their solutions in a variety of ways, including using logical reasoning without formal mathematical notation, using graph theory (e.g., digraphs), or using Dilworth's Theorem and reasoning about chains and antichains. The three official solutions provide examples of these approaches. Award **5 points** for any complete and correct solution, and **4 points** for a solution that is almost complete and correct, but has a minor gap or a key step that is not well explained.

Note: Award **1 point** of partial credit if the student uses the Pigeonhole Principle to note that if there are 26 mathematicians, then there must either be a group of 6 mathematicians or 6 groups of mathematicians, but doesn't otherwise make significant constructive progress.

Note: If a student proves the weaker result that there must be 6 mathematicians such that each pair was asleep at the same time, but not that all 6 mathematicians were asleep at the same time, award **4 points**. Solutions using Dilworth's Theorem to show that there is no antichain of size 6 fall into this category.

Note: Students must make use of the fact that the mathematicians sleep in intervals. For example, to have a valid solution using the Erdős-Szekeres Theorem, students must use the version that applies to *intervals* instead of *sequences*. If the student uses the version that applies to sequences, they should get a total score of **1 point** because of the use of the Pigeonhole Principle.

Note: Students cannot use $R(6,6)$ to solve the problem because $R(6,6) > 26$, even though the exact numerical value is not currently known. This incorrect approach should receive **0 points**.

Note: Do not deduct any points if the student doesn't consider the edge case in which two or more mathematicians wake up at the same time.

Problem 5/1/36:

1 point: Student shows that when $|b| \leq 2$, $f(x) \geq 0$ for all real x, so there is no solution to $f(f(x)+x) < 0$.

1 point: Student finds the four roots of $f(f(x)+x)$ in terms of the roots of $f(x)$.

3 points: Student provides a complete and correct analysis that includes all relevant cases. For submissions that are similar to the official solution, award **1 point** for each of the three cases/subcases in the official solution (Case 1, Case 2.1, Case 2.2). If the student just analyzes some specific numerical values of b for $|b| > 2$, award **1 point** for this section of the rubric.

Note: Some students did a bunch of algebraic steps that don't really lead anywhere. If the student's algebra doesn't reveal any of the possible values for the number of integers x that satisfy the inequality, give a score of **0 points**.

Note: If the student obtains the correct answer (all of 0,1,2 and no other values) but provides no explanation, award **1 point**.

Note: Award **4 points** for an otherwise correct solution in which the student shows that the solutions must be in intervals of length 1, but misses the case in which one of the intervals of length 1 has integer endpoints.

Note: Award at most **4 points** if the student doesn't recognize that there are two possibilities for the order of the roots from least to greatest.

Note: Award a total score of **1 point** if the student obtains the correct answer by experimenting with different values of b using the Desmos slider. Award **0 points** if the student uses the Desmos slider, but doesn't obtain all the possibilities for the number of integer solutions (e.g., the student doesn't recognize that some values of b have exactly one integer solution).