USA Mathematical Talent Search
Round 3 Grading Rubric
Year 31 — Academic Year 2019–2020
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GENERAL GUIDELINES

1. The grading rubric is designed to be simultaneously **specific** and **flexible**. For common solution methods, the rubric provides a specific allocation of points to ensure consistency across graders. Less common solution methods might not be captured closely by the rubric. For less common solution methods, consider the amount of constructive progress (including any specific intermediate results discussed in the rubric) and how far or close the student is to a complete solution when determining the score.

2. On **all** problems, the graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.

3. Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. Merely citing the name of a software package is not sufficient justification.

4. A student's justification needs to be rigorous and reasonably clear in order for the solution to merit **5 points**. If there is a meaningful gap in the student's argument or a key step is unclear, deduct points accordingly.

Problem 1/3/31:

Award **5 points** for the correct configuration of numbers. Withhold **1 point** for each incorrect entry.

Problem 2/3/31:

Note: Student solutions varied considerably in the clarity of the explanation, so we tended to evaluate solutions holistically when determining how many points to award. We've indicated below some specific things that were useful to include in a solution, but even if all of these elements were present, we deducted points if the explanation was unclear.

1 point: Student claims that if a square is guarded by three or more sentries, then there must be two sentries that attack each other.

2 points: Student proves the above claim by showing that if a square is guarded by three or more sentries, then there must be two sentries in the same row or two sentries in the same column (without a wall in between them), so those sentries would attack each other.

2 points: Student uses the pigeonhole principle to show that if there are more than 2000 sentries, then at least one square must be guarded by at least three sentries, in which case two of the sentries would attack each other.

Problem 3/3/31:

1 point: Student proves that $d_i - d_{i-1}$ must divide d_{i-1}. If a student proves a different but similarly useful result, award this point.

1 point: Student explains why 2 and 6 are the first two squarefree juicy numbers.

1 point: Student explains why 42 is the next squarefree juicy number.

1 point: Student explains why 1806 is the next squarefree juicy number.

1 point: Student explains why there are no squarefree juicy numbers greater than 1806.

Note: If the the student writes the correct answer with no explanation, award **1 point**.

Problem 4/3/31:

1 point: Student creates a useful figure based on the given information.

1 point: Student discusses a homothety centered at D with scale factor 2.

1 point: Student recognizes that $\overline{OT}$ is the midline of $\triangle I'ID$.

1 point: Student recognizes that $FT = GT$ if and only if T is also on the perpendicular bisector of $\overline{FG}$.

1 point: Student recognizes that since $\overline{OT} \parallel \overline{ID}$, we have $FT = GT$ if and only if $\overline{ID} \perp \overline{FG}$.

Note: If a student only proved one direction of the if-and-only-if statement, we typically awarded **3 points**. However, if the student's argument was very easily reversible and it appeared that the student simply overlooked this, we typically awarded **4 points**.

Note: Some students did bashing solutions, which in most cases weren't too tedious. We tended to grade these solutions holistically. If a student didn't include a diagram, we typically awarded at most **4 points**.

Problem 5/3/31:

1 point: Student shows that f is an odd function.

1 point: Student shows that $f(\pi) = 0$.

2 point: Student shows that $f(\pi - x) = f(x)$. Award **1 point** for significant constructive progress towards this result.

Note: A common error was the following: "$f(\pi - x)^2 = f(x)^2$, so either $f(\pi - x) = f(x)$ or $f(\pi - x) = -f(x)$. If f$(\pi - x) = -f(x)$, then $f(\pi + x) = f(x)$, which contradicts the condition that the minimum period is 2π. Therefore, $f(\pi - x) = f(x)$." The flaw is that the period of 2π does **not** tell us that for all x, we have $f(x) \neq f(x + \pi)$. We just know that there is at least one x such that $f(x) \neq f(x + \pi)$. Students who made this error did not receive any credit for this part of the solution.

1 point: Student completes the proof and shows that $\left|f\left(\frac{\pi}{2}\right)\right| \geq f(x)$.