USA Mathematical Talent Search

Round 2 Grading Rubric

Year 29 — Academic Year 2017–2018

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NOTE TO GRADERS: The grading rubric is designed to be simultaneously **specific** and **flexible**. For common solution methods, the rubric provides a specific allocation of points to ensure consistency across graders. Less common solution methods might not be captured closely by the rubric. For less common solution methods, consider the amount of constructive progress (including any specific intermediate results discussed in the rubric) and how far or close the student is to a complete solution when determining the score.

IMPORTANT NOTE: On **all** problems, the graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.

IMPORTANT NOTE: Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. Merely citing the name of a software package is not sufficient justification.

Problem 1/2/29:

Award **5 points** for the correct configuration of letters. No justification is required. Withhold **1 point** for each incorrect entry.

Problem 2/2/29:

1 point: Student recognizes that once the sequence has passed b^{k-1}, it will increase by k each term until it passes b^k.

1 point: Student recognizes that if b^{r-1} is in the sequence, b^r will be in the sequence if and only if $b^r - b^{r-1}$ is a multiple of r.

1 point: Student recognizes that we are looking for the first power of b such that b^r is not congruent to $b^{r-1} \pmod{r}$.

2 points: Student applies the above reasoning to the case of $b = 2521$. Award **1 point** (out of these 2 points) if the student makes a minor error in this step that leads to an incorrect answer.

Problem 3/2/29:

5 points: Student proposes and provides a complete and clear explanation of a valid strategy for determining the representative using a tournament with 10 rounds. Award a total score of **4 points** if there is a relatively minor but non-trivial gap in the student's justification (e.g., the student doesn't account for the possibility of a tie). Fewer points should be awarded if the gap is more substantial.

1 point: Student proposes a plausibly useful and systematic (but ultimately incorrect) swapping strategy, and provides at least some correct analysis of how (in some cases) the team will be able to select its tournament representative. Additional points may be awarded if there is a repairable error or gap in the student's analysis that if fixed, would lead to a complete and correct solution.

Problem 4/2/29:

Part (a) is worth **3 points** and part (b) is worth **2 points**.

Part (a):

3 points: Complete and correct solution.

2 points: Correct except for a non-trivial gap or flaw in the student's argument (but the solution is more than halfway to a complete and correct solution). If the student gets to roughly the point in the official solution "So to complete the induction we need to show that

$$m + \frac{b-a}{d} - 1 < b - a,\text{''}$$

then the solution is strong enough to receive **2 points**.

1 point: Student sets up and begins to implement a viable solution strategy, but there are major errors or gaps in the student's argument. For students who use the induction argument, students must correctly analyze the base case and make at least some progress with the inductive step.

0 points: No meaningful and significant constructive progress.

Note: Some students did not fully consider the case in which Zan makes intermediate reductions. Students who asserted without proof that Zan will stop faster if there are intermediate reductions should receive a maximum of **2 points** on this part and students who did not consider intermediate reductions at all should received a maximum of **1 point**.

Part (b):

1 point: Student proves that $\frac{n-1}{n} < \frac{a}{b}$.

1 point: Student proves that $\frac{a}{b} \le \frac{n}{n+1}$.

Note: It is far easier to show $\frac{a}{b} \le \frac{n}{n+1}$ than $\frac{n-1}{n} < \frac{a}{b}$, but the former is still worth a point. That said, if the student's proof of $\frac{n-1}{n} < \frac{a}{b}$ is correct except for a small error, a score of **1 point** for part (b) should be awarded, even if the student doesn't prove that $\frac{a}{b} \le \frac{n}{n+1}$.

Problem 5/2/29:

1 point: Correct analysis of the $n = 5$ case (i.e., there are at most 7 independent segments). The $1 \le n \le 4$ cases are sufficiently easy that no points were awarded for those cases.

2 points: Correct analysis of the maximum number of independent segments if the convex hull is a triangle. To receive **1 point** of partial credit, it sufficient to provide a correct analysis of any of the following: (1) the case in which AX and BY cross, (2) the case in which AX and BY do not cross, (3) the maximum number of independent segments if AX, BX, and CX are all independent, or (4) the maximum number of independent segments if there is no interior point X such that AX, BX, and CX are all independent.

Note: If a student has a solution that is correct with the exception that they omit the case that the convex hull is a triangle, they should receive a total score of **4 points** on this problem.

2 points: Correct analysis of the maximum number of independent segments if the convex hull is not a triangle. Either of the following is sufficient for **1 point** of partial credit: (1) recognizing that if both endpoints of an independent segment are vertices of H then the segment must be a side of H, or (2) a correct analysis of the number of "doubles."

Note: The specific results mentioned above are not intended to be the only useful results that are worthy of partial credit. Award appropriate partial credit for other useful results, especially if the student uses a different method.