USA Mathematical Talent Search

Round 1 Grading Rubric

Year 29 — Academic Year 2017–2018

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NOTE TO GRADERS: The grading rubric is designed to be simultaneously **specific** and **flexible**. For common solution methods, the rubric provides a specific allocation of points to ensure consistency across graders. Less common solution methods might not be captured closely by the rubric. For less common solution methods, consider the amount of constructive progress (including any specific intermediate results discussed in the rubric) and how far or close the student is to a complete solution when determining the score.

IMPORTANT NOTE: On **all** problems, the graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.

IMPORTANT NOTE: Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. Merely citing the name of a software package is not sufficient justification.

Problem 1/1/29:

Award **5 points** for the correct configuration of numbers. No justification is required. Withhold **1 point** for each incorrect entry.

Problem 2/1/29:

1 point: Student introduces and clearly defines a relevant set of variables.

2 points: Student sets up a valid inequality similar to $\frac{1+5n}{n+1} \geq r$. Award **1 point** of partial credit if the student's inequality has the *number of pre-barfing rides* as a variable.

2 points: Student performs a series of algebraic manipulations to show that the number of future 5-star rides needed does not depend on the number of previous rides (e.g., $n \geq \frac{r-1}{5-r}$).

Note: Students should receive full credit **(5 points)** if they reason from the inequality $\frac{1+5n}{n+1} \geq r$ that n only depends on r, even if they don't isolate n.

Note: We did not penalize students who used equations instead of inequalities.

Problem 3/1/29:

5 points: Student demonstrates a way to combine two polygons to get all three of the desired shapes (triangle, convex quadrilateral, convex pentagon). The student doesn't need to include a lot of supporting explanation to get full credit as long as it is easy to see that the constructions work. However, if the student's work is unclear, deduct at least **1 point**.

Each of the following two issues warrants a **1 point** deduction: (1) the student does not include a diagram, (2) the student does not show that the three relevant points are collinear when forming the quadrilateral.

2 points: Student demonstrates a way to combine two polygons to get two out of three of the desired shapes.

0 points: Student demonstrates a way to combine two polygons to get one out of three of the desired shapes, or the student argues that the answer to the problem is "no."

Exception: if the student answers "no," but is still able to combine two polygons to get two out of three of the desired shapes, the student should get **2 points** for their constructive progress.

Problem 4/1/29:

1 point: Student recognizes that the two plausibly optimal strategies for the first player involve (1) putting an 8 in the top row or (2) putting a 1 in the bottom row during their first turn.

1 point: Student provides a rigorous and accurate analysis of what the second player should do during their first turn (i.e., put the lowest remaining number in the top row).

3 points: Student argues (with justification) that if the first player puts a 1 in the bottom row on their first turn and then puts the lowest remaining number (2 or 3) in the bottom row on their second turn, then the first player is guaranteed to win.

Note (see revision below): Some students suggested that the first player should put a 1 in the bottom row, the second player should put a 2 in the top row, and then the first player should put an 8 in the top row. This is a winning strategy for the first player, but it is still possible for the first player to lose after placing the 8 in the top row (on the second turn) if they make suboptimal decisions on subsequent turns. Thus, this strategy is not optimal. Students who provide this solution should receive a total score of **3 points** on this problem.

Revision to note: In reviewing the protests for Round 1 Problem 4, one student made the argument that providing the maximum-scoring strategy is not necessary for determining the winner of play with perfect strategy (and this is correct). As a result, we are adjusting the grading criteria for this problem.

We will now award a full **5 points** for a proof that the first player has a winning strategy. To receive the full **5 points**, a strategy must be given that guarantees a win for the first player, and a proof must be given that the strategy guarantees victory against all possible strategies that the second player might employ.

Problem 5/1/29:

2 points: Student proposes a set S that satisfies all the conditions in the problem.

3 points: Student explains how to express all integers n as the sum of the elements of a unique nonempty, finite subset of S. Award partial credit as appropriate if there are gaps in the student's explanation. Award **1 point** of partial credit if the student shows how to express all integers n as the sum of the elements of a finite subset of S, but does not show that there is a unique way to express each integer. Award **2 points** of partial credit if the student makes significant constructive progress towards proving that there is a unique way to express each integer, or if the solution is completely correct except for a non-trivial step that needs additional justification.

Note: A student should receive a score of **1 point** if they propose a set such as $1, -2, 4, -8, 16, \ldots$ that can uniquely make almost every integer *with a finite number of exceptions*. Students should receive an additional point **(2 points total)** if they state (with explanation) that the set of integers that cannot be expressed as the sum of the elements of a unique nonempty, finite subset of the proposed set.

Note: Award **1 point** for a significant useful observation, even if the student incorrectly argues that a set S meeting the criteria in the problem does not exist. An example of such a useful observation would be showing that some member of S_0 (defined as the subset of S that sums to 0) must intersect infinitely many S_i. To get this point, it is not sufficient to say that S_0 must intersect every other S_i.