USA Mathematical Talent Search

Round 2 Grading Rubric

Year 27 — Academic Year 2015–2016

www.usamts.org

IMPORTANT NOTE: On **all** problems, the graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.

IMPORTANT NOTE: Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. Merely citing the name of a software package is not sufficient justification.

Problem 1/2/27:

Award **5 points** for the correct configuration of numbers. No justification is required. Withhold **1 point** for each number that is placed incorrectly.

Problem 2/2/27:

Any correct example, including a description of the two distinct polyhedra (a clear written description, a standard name (e.g., octahedron), or a picture is sufficient), the nets of these polyhedra, and how the nets can be divided into the same two pairs of pieces should receive a score of **5 points**. Award **4 points** if the example is correct, but unclear.

Note: The problem did not specify that both polyhedra should be convex, so if a student provides an example in which the two polyhedra are identical except that one of them has a vertex that is indented to make it concave, the student should receive **5 points** if the justification is complete.

If the student provides nets for two distinct polyhedra **that CAN be divided into the same two pairs of pieces**, but the student proposes an incorrect way of dividing the nets into pieces (or does not propose a division), award **3 points**.

If the student incorrectly answers **NO**, award up to **1 point**. Only award **1 point** if the students provides a thoughtful (albeit incorrect) explanation that demonstrates deep understanding of the problem.

Note: A completely incorrect example is worth **0 points**.

Problem 3/2/27:

Students using the method in **Solution 1** (see official solutions) or another inductive argument should receive points as follows:

1 point: Student proposes a key claim that they wish to prove by induction.

3 points: Student proves the claim by induction. Of these points, **1 point** should be awarded for demonstrating the base case and **2 points** should be awarded for proving the inductive step. Award **1 point** (out of these 2 points) if the student makes meaningful constructive progress towards proving the inductive step.

1 point: Student applies the key claim to prove that the statement in the problem is true.

Students using the method in **Solution 2** (see official solutions) should receive points as follows:

1 point: Student proposes that if an n-sided die is rolled repeatedly, the probability that the first repeat outcome will occur on the $(k+1)$st roll is equal to $\frac{1}{n}\frac{k \cdot k! \cdot \binom{n}{k}}{n^k}$ (assuming $0 \leq k \leq n$).

2 points: Student proves the preceding claim. Award **1 point** (out of these 2 points) if the student makes meaningful constructive progress towards proving this claim.

1 point: Student recognizes that the first repeat outcome must occur on or before the $(n+1)$st roll.

1 point: Student uses the preceding facts to complete the proof that the statement in the problem is true.

Problem 4/2/27:

2 points: Student shows **with justification** that both $P(x) = cx$ and $P(x) = c$ work, where c is a constant (**1 point** for each). It is sufficient to plug in cx and c into $P(a)$ and $P(a+b)$ and do the relevant calculations to show that $P(a+b) - P(a)$ is a multiple of $P(a)$. Each of these points should be awarded even if the student says incorrectly that there are other functions that work. Award **1 point** total if the student says that both $P(x) = cx$ and $P(x) = c$ (but no other values of $P(x)$) work, but the student does not provide acceptable justification of this claim.

3 points: Student shows that no other other values of $P(x)$ work. **1 point** or **2 points** of partial credit may be awarded based on the extent of the student's constructive progress. Each of the following two intermediate results **with justification** is worth **1 point**. (1) Student shows that if $P(x)$ has degree n, $P(x)$ has no lower-degree terms. (2) Student shows that $P(x+b) - P(b) = P(x)$.

Problem 5/2/27:

2 points: Student shows with justification that for any positive even value of n, it is possible to have $\frac{3}{2}n^2 - 2$ elements in S. A construction for the general case, with an explanation of why S has $\frac{3}{2}n^2 - 2$ elements, is sufficient. To receive **1 point** of partial credit, student needs to (a) get the correct answer with some justification, or (b) make significant constructive progress towards a valid construction for the general case (e.g., a student creates a construction, but there are minor errors that lead to an incorrect answer, a student creates a valid construction for a slightly higher number of elements in S). To receive partial credit, it is NOT sufficient to show that $\frac{3}{2}n^2 - 2$ works only for a specific value of n.

3 points: Student shows with justification that S cannot have less than $\frac{3}{2}n^2 - 2$ elements. Award appropriate partial credit for significant constructive progress towards this result. If a student simply says that if we unshade any of the squares in S in their particular construction (as opposed to any possible S), then the revised grid does not satisfy all the conditions in the problem, the student should NOT receive any partial credit.