Round 3 Grading Criteria

IMPORTANT NOTE: On **all** problems, the graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.

IMPORTANT NOTE: Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem. Any solutions relying on computer methods should include the source code or specify the function call(s) (with arguments) used in a computer algebra system. Merely citing the name of a software package is not sufficient justification.

Problem 1/3/26:

5 points should be awarded for the correct configuration of numbers. No justification is required. Withhold a point for each number that is placed incorrectly.

If the configuration is not fully correct, you should specifically point out at least one error in your comments.

Problem 2/3/26:

1 point: Student divides the figure into triangles (or divides the figure in other helpful ways) to facilitate the calculation of the area of the shaded and non-shaded regions.

2 points: Student recognizes with justification that the triangle with area z (see official solution) is congruent to (and therefore has the same area as) the triangle with area x. One of the two points should be awarded if the justification is missing or incomplete. If the student divided the figure into other types of regions, appropriate credit should be awarded for the equivalent analysis.

2 points: Student recognizes with justification that the shaded and non-shaded regions have equal area and student obtains the correct answer of 1/2.

Note: Alternatively, some students used trigonometry to calculate the areas of the shaded and non-shaded regions. Correct solutions using this approach received full credit, and incorrect attempts received partial credit based on the amount of constructive progress.

Problem 3/3/26:

1 point: Student recognizes with proof that the sequence $a_1, a_2, a_3, \ldots$ is a non-decreasing sequence. In some cases, one point was awarded for other constructive progress.

4 points: Student recognizes with justification that $\left(\frac{q}{q+1}\right)\log_n(a_n) \le \log_2(a_2) \le \left(\frac{q+1}{q}\right)\log_n(a_n)$ or establishes a similar type of bound on $\log_2(a_2)$ AND student explains how this proves that $\log_{2015}(a_{2015}) = \log_{2014}(a_{2014})$. Any student who does not explain how the bound on $\log_2(a_2)$ proves the key claim in the problem should receive three of the four points.

Student methods for proving this claim varied considerably, and partial credit should be awarded based on the amount of constructive progress. For students who followed the official solution, partial credit should be awarded for obtaining (with justification) the following intermediate results.

One point should be awarded for obtaining $n^q < 2^p \leq n^{q+1}$ or $(a_n)^q \leq (a_2)^p \leq (a_n)^{q+1}$.

A second point should be awarded for obtaining $\frac{q \log_2(a_n)}{(q+1)\log_2(n)} \leq \log_2(\lambda) \leq \frac{(q+1)\log_2(a_n)}{q \log_2(n)}$.

A third point should be awarded for obtaining $\left(\frac{q}{q+1}\right) \log_n(a_n) \leq \log_2(a_2) \leq \left(\frac{q+1}{q}\right) \log_n(a_n)$.

Note: If the student states the hypothesis with some non-rigorous reasoning that $\log_{2015}(a_{2015}) = \log_{2014}(a_{2014})$, and thus, that $\log_{2015}(a_{2015}) - \log_{2014}(a_{2014}) = 0$, one point should be awarded.

Problem 4/3/26:

1 point: Student proves that for every two consecutive entries on the circle, there must be a prime p so that p divides the product of those entries fewer times than it divides n.

1 point: Student proves that for every two non-consecutive entries on the circle, for every prime p, p divides the product of those entries at least as many times as p divides n.

1 point: Student recognizes that for each entry of the circle, you can identify a prime p that divides the product of that entry and the subsequent entry fewer times than it divides n, and then proves that each prime p can appear at most twice in this fashion. Moreover, if p appears twice, it must appear in consecutive entries.

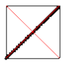
1 point: Student proves that if a prime p appears in consecutive entries of the circle, then p^2 divides n.

1 point: Student uses these facts to construct a prime factorization and recognizes that n is minimized when $n = 2^2 3^2 5^2 7^2 11^1 = 485{,}100$.

Note: If a student does not recognize that each prime can be used twice and obtains the answer that $n = 2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19 \times 23$, a maximum of two points should be awarded.

Problem 5/3/26:

5 points: Student provides a rigorous argument that the number of triangles *must be* a multiple of 4. Partial credit may be awarded based on the extent of the constructive progress. One acceptable argument was to demonstrate that the triangles in the tiling must form a set of cycles, each of which must be a multiple of 4.

Note: If the student does not provide a rigorous argument, but provides an example of a tiling in which the number of triangles is a multiple of 4, one point should be awarded. One example is . One additional point of partial credit should be awarded if the student sketches out a non-rigorous intuition for why the statement in the problem is true.