Round 1 Grading Criteria

IMPORTANT NOTE: On **all** problems, the graders have the discretion to deduct one additional point for a solution that is poorly written and/or hard to follow.

IMPORTANT NOTE: Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem. Any solutions relying on computer methods should include source code or specify the function call(s) (with arguments) used in a computer algebra system. Merely citing the name of a software package is not sufficient justification.

Problem 1/1/26:

Five points should be awarded for the correct configuration. No justification is required. Withhold a point for each unit square that is in an incorrect region.

Problem 2/1/26:

2 points: Student recognizes (with justification) that z must be 2. Award only one of the two points if the justification is incomplete, missing, or incorrect.

1 point: Student recognizes (with justification) that $x - y$ is even and prime, so $x - y = 2$ and $x = y + 2$.

1 point: Student recognizes that when $z = 2$, the triple (7, 5, 2) satisfies all the criteria in the problem.

1 point: Student recognizes (with justification) that (7, 5, 2) is the only triple that satisfies all the criteria in the problem.

Problem 3/1/26:

Note: Two interpretations were accepted: (1) the standard interpretation in which A is to the left (but not necessarily directly to the left) of B, and (2) an alternate interpretation in which A is (necessarily) directly to the left of B.

Part (a) is worth 2 points and part (b) is worth 3 points.

Part (a)

1 point: Statement of the Lemma that if we have n people whose heights are a, $a + 5$, …, $a + (n - 1)5$ centimeters for some a, then there are F_{n+1} ways to line them up in almost-order, where F_k is the k^{th} Fibonacci number, defined by $F_0 = 0$, $F_1 = 1$, and $F_k = F_{k-1} + F_{k-2}$ for all $k \geq 2$. Student must also apply the Lemma to the given set of 10 people, which shows that there are 89 possible almost-orderings.

For students who used the alternate interpretation, there should be 2^{n-1} almost-orderings; for the given set of 10 people, this yields 512 almost-orderings.

1 point: Proof of the Lemma.

Part (b)

3 points: Correct division of the problem into reasonable cases and correct analysis (including justification) of all cases, including the correct answer of 23,322 almost orderings. Partial credit (one or two points) may be awarded depending on the extent of the student's progress.

For students who used the alternate interpretation, the correct answer is $9 \times 2^{17} =$ 1,179,648 almost-orderings. Similarly, partial credit (one or two points) may be awarded depending on the extent of the student's constructive progress.

Problem 4/1/26:

Student solutions varied considerably. Solutions based on the official solution were awarded points as follows:

1 point: Student recognizes the fact that because A, B, C, D are vertices of a regular n-gon, the arcs AD, DB, and BC (and similarly, the arcs AF, FB, and BE) all have lengths that are positive multiples of s, where $s = 2\pi / n$.

1 point: Student recognizes that if x, y, and z are the lengths of arcs CF, FD, and DE, respectively, it is sufficient to show that x, y, and z are each half-integer multiples of s.

1 point: Student recognizes (with justification) that $x + y$ and $y + z$ are integer multiples of s.

1 point: Student recognizes (with justification) that FC + CB + BE + ED + DA +AF = 2π and explains why this means that $x + z$ is an integer multiple of s.

1 point: One point should be awarded if the student recognizes that $x = \frac{(x+y)+(x+z)-(y+z)}{2}$ (plus the corresponding equations for y and z) and shows how these facts can be used to prove that x, y, and z are each half-integer multiples of s.

The following partial credit was awarded for constructive progress.

1 point: Student draws a correct diagram.

2 points: Student shows that the construction of the $2n$-gon is possible for a specific case ($n = 6$). Students did not receive a third point for the correct diagram.

Problem 5/1/26:

2 points: Student recognizes (with justification, such as an inductive argument) that for all $n \geq 3$, $a_n = x$ if n is odd and $a_n = y$ if n is even; since $a_{2014} = 2015$, the sequence is a_0, a_1, 5, x, 2015, x, 2015, x, … Award only one of the two points if the justification is incomplete, missing, or incorrect.

3 points: Student shows (with a case-by-case analysis) that $a_{2015} = x$ can be 2015 or any even integer greater than 2. One or two points may be awarded if the student correctly analyzes a portion of the cases. Two points should be awarded if a majority of the necessary cases are analyzed correctly and all necessary cases are identified. One point should be awarded if a minority of the necessary cases are analyzed correctly or if a majority of the necessary cases are analyzed correctly, but some necessary cases are not identified.