USA Mathematical Talent Search

Round 3 Grading Criteria

Year 25 — Academic Year 2013–2014

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IMPORTANT NOTE: On **all** problems, the graders have the discretion to deduct 1 additional point for a solution that is poorly written.

Problem 1/3/25:

5 points for a correct configuration. Withhold a point for each cell that was filled with an incorrect number.

If a configuration is not fully correct, you should specifically point out at least one error in your comments. A correct proof of uniqueness should be commended in the comments.

Problem 2/3/25:

Most students solved this problem by methods that are similar to the provided solution. Points should be awarded as follows:

1 point: Student recognizes that there is degree 5 periodicity in the recurrence. Award this point even if there is no meaningful justification of this fact.

2 points: Student successfully argues why the degree 5 periodicity holds. For a student who merely shows that $a_6 = a_1$, award only one of the two points for this part. If a student shows that $a_6 = a_1$ and that $a_7 = a_2$, but does not explicitly state that this is sufficient to show periodicity (as each term only depends on the previous two terms), award 2 points and make a comment that the student should point out why this is sufficent.

1 point: Student recognizes that 3 is the maximum value. Award this point even if the student has no meaningful justification of this fact.

1 point: Student successfully justifies why the maximum value of $a_{2014} = a_4$ is 3.

Appropriate credit should be awarded for full and partial solutions that use other correct approaches to the problem. Any solutions relying on computer methods should include source code or specify the function call(s) (with arguments) used in a computer algebra system. Merely citing the name of a software package is not sufficient justification.

Problem 3/3/25:

There were a wide variety of solutions to this problem. Many of the methods that students used to approach this problem could lead to a correct proof. Students do not need to prove that the figure is symmetric about line A_6A_{16} in order to use this fact in their proof.

Award **1 point** for any response that recognizes that, due to the symmetry across line A_6A_{16}, that it suffices to show that three lines are concurrent.

The remaining **4 points** can be awarded for reasonable progress towards a solution.

Graders should keep the following in mind when grading:

- Feel free to withhold a point for any solution that is hard to follow. This may also include solutions that are missing diagrams, especially if the student refers to points or lines that are not drawn in the original figure.
- Geometric claims should be justified. Students who describe figures as congruent or lines as concurrent should provide at least some evidence for these claims. If you are deducting points for this reason, please be clear, and make a comment along the lines of, "You have not sufficiently proven why $\triangle ABC \cong \triangle DEF$."
- Be especially wary of unsupported claims, such as "We can easily derive that. . . ." Specifically point out such weaknesses in the comments.
- If students cite the use of a computer method for evaluating expressions, such as trigonometric expressions, use your discretion about whether to require the specific function call or source code. As a rule of thumb, if a student is merely evaluating an expression, such as $\cos(36°)$, no further justification is necessary. However, if a student has a trigonometric equation and is solving for a variable using computer software, then the student should include an explanation of the method and not just the name of the software.

Problem 4/3/25:

Divide the points up as follows:

2 points: Showing that if a number is not a multiple of 16, then it can not be a cut size.

3 points: Showing that if a number is a multiple of 16, then it must be a cut size. This can be further subdivided, when appropriate as:

- **1 point:** The student has an analysis of partial sums.
- **1 point:** The student attempts to make a pigeonhole argument.
- **1 point:** Execution of the strategy.

A student who says that cut sizes are multiples of 16 with no meaningful justification can receive at most one point. A student who states that cut sizes have size exactly 16 with no meaningful justification shall receive 0 points.

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Problem 5/3/25:

Part (a) is worth 2 points and part (b) is worth 3 points.

For both parts, the essential idea is that if we need to represent a number n with the set of digits $\{-1, 0, 1, 4a+2\}$, then the units digit of the number is determined by n's residue modulo 4. If that unit's digit is d, then to compute the remaining digits, it suffices to find a representation of $\frac{n-d}{4}$, and append d to the end of this. When this process always terminates, we get a 4-basis. When there is a number for which the process enters an infinite loop, we do not get a 4-basis.

Here are points of partial credit for (a):

- 2 points only for a complete and correct proof.
- 1 point out of 2 if the infinite family $a \equiv 1 \pmod 3$ is suggested to work (digits are $\{-1, 0, 1, 12a+6\}$), but the proof is not given or is incorrect.
- 1 point out of 2 if the above points do not apply, anything about the algorithm discussed above ($n \to \frac{n-d}{4}$) is mentioned, particularly in regard to needing an infinite loop to show something is not a 4-basis. No proof is required to get this point.

There are several ways to do (b). One can use induction, or one can use an digit-flipping algorithm beginning from a known representation using a digit set such as $\{0, 1, 2, 3\}$ or $\{-1, 0, 1, 2\}$. All these methods revolve around showing a digit set resembling $\{-1, 0, 1, \pm 4^k \pm 2\}$ works; there do not seem to be any contestants who solved (b) without using powers of 4.

- 3 points only for a complete and correct proof.
- 2 points out of 3 for a mostly correct proof with omissions. For example, if the solution requires that $\{-1, 0, 1, 2\}$ and/or $\{-2, -1, 0, 1\}$ is a 4-basis but handwaves this, deduct a point. This is not well-known.
- 2 points out of 3 if powers of 4 plus or minus 2 are suggested for the fourth digit **and** the student correctly proves that $\{-1, 0, 1, 2\}$ and/or $\{-2, -1, 0, 1\}$ is a 4-basis, but not much more progress is made.
- 1 point out of 3 if powers of 4 plus or minus 2 are suggested for the fourth digit.

Some solutions may do things very differently from how the above list expects. Give partial credit as you see fit. However, you should almost never give a positive score for (b) if the solution is incomplete and no mention is made of powers of 4.