USA Mathematical Talent Search

Round 1 Grading Criteria

Year 25 — Academic Year 2013–2014

www.usamts.org

IMPORTANT NOTE: On **all** problems, the graders have the discretion to deduct 1 additional point for a solution that is poorly written.

Problem 1/1/24:

Break the points on this problem into two parts:

- 2/2. Show that every possible code cannot be attempted with fewer than 29 button presses.
- 3/3. Demonstrate that 29 button presses are sufficient to try every possible code. Giving an explicit sequence of 29 letters that contains every three letter code is enough to get full credit on this part.

As usual, a correct solution with a method that does not neatly divide in this way is worth 5 points.

Award 3 total points if a correct solution uses a theorem about de Bruijn sequences without citation.

Problem 2/1/24:

5 points for a correct configuration. Award 4 total points if the entire configuration is right save for one or two errors. Award 3 total points if the configuration is not correct, but the solution has the correct residue classes modulo 3 in each grid cell.

If a configuration is not fully correct, you should specifically point out at least one error in your comments. A correct proof of uniqueness should be commended in the comments.

Problem 3/1/24:

For this problem you will find both constructive solutions and existence solutions. Generally split the points on this problem into three pieces:

- 1/1. The answer "yes" with some nontrivial understanding of why.
- 2/2. An explicit construction or algorithm (toward an existence proof) that gives a correct answer.
- 2/2. The proof that the answer is correct.

Almost all correct solutions rely on placing sets of some "minimal distance" on top of one another. The most efficient submissions will follow the given solution.

The single key idea is that in this problem, if the minimal distance is like $\frac{1}{n}$ then the number of points we should be able to fit is proportional to n, and we can build an algorithm that hits these bounds for all n equal to a power of 2 (for example). Almost all solutions that answer "no" fail to see this, so should be given 0 points. However, a "no" solution that has some interesting insight into the problem may be awarded 1 point.

One common error is the misconception that all series of the form

$$\frac{1}{1} \pm \frac{1}{2} \pm \frac{1}{3} \pm \frac{1}{4} \pm \frac{1}{5} \pm \cdots$$

will diverge. These solutions will almost always be worth 0 points.

Problem 4/1/24:

Use the following general guidelines:

- 1/5: Demonstrate some insight with an arithmetic sequence that follows the rules.
- 2/5: Prove that if n is bunny-unfriendly then Bunbury will choose a d that is divisible by all prime factors of n less than 2013.
- 4/5: Give the correct answer for the maximum bunny-unfriendly integer along with the proof that if n is bunny-friendly then Bunbury will choose a d that is divisible by all prime factors of n less than 2013.
- 5/5: Give the correct answer for the maximum bunny-unfriendly integer along with the proof that if n is bunny-friendly then Bunbury will choose a d that is divisible by all prime factors of n less than 2013 and also control over the prime factors of n that are greater than 2013.

You will see many different attacks for this problem. However, a central theme is almost always casework on the profile of the primes dividing n.

You may see a few other novel and unpredicted approaches to this problem. For example, brute force is a possibility after some very clever analysis. Award points appropriately.

Problem 5/1/24:

It helps to consider this problem in these terms: For any triangle, there is some function f taking a point X in the triangle and giving $f(X)$, the minimal area that Niki can produce. This is a continuous, but piecewise function—it changes in an important way at the medial triangle of $\triangle ABC$. If Niki is playing optimally, the value S is equal to $f(X)$ for the point X that Kyle chooses. Therefore Kyle is trying to find the maximum value for f on Niki's triangle.

Award points on this problem as follows.

- 1/1. Stating each player's optimal strategy including the final answer.
- 1/1. There is nontrivial logic to this problem. Award one point for setting up the logic in the solution correctly and completely.

- 2/2. Maximizing f inside the medial triangle.
- 1/1. Proving that the maximum for f lies inside the medial triangle (the values outside are smaller, that is).

Notes:

The choice of triangle is irrelevant. However, it is also irrelevant whether the student states this point, as long as all triangles of area 1 are considered.

Barycentric coordinates can be very useful on this problem. Students are free to use barycentric coordinates without citation.