# USA Mathematical Talent Search

Round 3 Grading Criteria

Year 24 — Academic Year 2012–2013

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**IMPORTANT NOTE:** On **all** problems, the graders have the discretion to deduct 1 additional point for a solution that is poorly written.

**Problem 1/3/24:**

A correct solution, with or without explanation, is worth 5 points. If one number was either missing or incorrect (meaning a duplicate), award 4 points. If one pair of numbers was swapped, award 3 points. More errors than this are worth less than 3 points, and usually 0 points. A misinterpretation of the rules, especially ignoring the no-touch rule, is worth 0 points.

An answer given in coordinates in which the contestant seems to have had a correct solution but transcribed the coordinates incorrectly (up to 3 shifted in incorrect $x$ or $y$ direction) is worth 2 points.

**Problem 2/2/24:**

Stating the correct answer $\frac{17}{32}$ is worth 1 point.

Giving a correct construction for $\frac{17}{32}$ is worth 1 point.

The remaining 3 points come from proving that no higher probability can be achieved. A computer-based brute force solution should earn all three points if it is correct and complete. Give only 1 point if there is handwaving, e.g. saying that having two zeroes is clearly the best way of balancing two competing constraints.

**Problem 3/2/24:**

There are a lot of different ways to solve this problem. Any fully correct solution is worth 5 points. Below are point totals awarded for some common mistakes. Solutions which make different mistakes should be awarded a score at the grader's judgment, aiming for consistency with the below guides.

- Solutions based entirely on false synthetic geometry assumptions are worth 0 points.
- Some solutions obtain an empirical measurement of the angle, often through a precise diagram. If this obtains the correct answer, award 1 point.
- If a complete solution requires decimal approximations (often of trigonometric functions), award 2 points.
- If a solution reaches a trigonometric equation in a variable, verifies it has a certain solution, then derives the answer from this without showing the trigonometric equation has a unique solution, award 2 points. If the trigonometric equation is particularly simple and showing uniqueness is not hard, award 3 points.

**Problem 4/2/24:**

The rubric below anticipates a solution like the official one. Note there are many different ways of stating the same arguments; in particular many solutions get by without mentioning base expansions.

1. Obtaining an equivalent of the equation $\lfloor a_k \rfloor - \lfloor a_{k-1} \rfloor = d_{k-1}$ where the base-$m$ expansion of $s$ is $0.d_1d_2d_3\ldots$. (This is marked as equation (1) in the official solutions.) Worth 3 points in total. Award 1 point if either the equation $\{a_k\} = \{ma_{k-1}\}$ is reached or if base-$m$ expansions are considered.

2. Finishing the proof from the above, which mostly involves working with periodic sequences. Worth 2 points in total.

As usual, a correct solution with a method that does not neatly divide in this way is worth 5 points.

**Problem 5/2/24:**

The solution to this problem has three parts:

1. Showing that $p_n = \pm q_n$, where $p_n, q_n$ are the leading coefficients of $P(x), Q(x)$ respectively. Worth 1 point in total.

2. Dealing with case 1 in the official solutions, where $p_n = q_n$ and all such $P(x)$ are quadratic. Worth 2 points in total. Give 1 point if they do the argument showing that the degree of $P(x)$ must be 2 (even if they erroneously claim that degree 2 polynomials fail as well).

3. Dealing with case 2 in the official solution, where $p_n = -q_n$ and $P(x) = -Q(x)$. Worth 2 points in total.

As the remark in the official solution notes, a solution $P, Q$ exists for both cases, so any correct solution must deal with both.

If none of the above points are awarded but the student notices that $P(-x) = P(x) + x$ is equivalent to the only term with odd exponent being $-\frac{x}{2}$ in $P(x)$, 1 total point may be awarded.