USA Mathematical Talent Search

Round 1 Grading Criteria

Year 24 — Academic Year 2012–2013

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IMPORTANT NOTE: On **all** problems, the graders have the discretion to deduct 1 additional point for a solution that is poorly written.

Problem 1/1/24:

5 points for a solution that gets the answer of 33 through a valid method of counting. If a solution merely lists out all 33 possibilities without any attempt to show their list is comprehensive (i.e. no accompanying program or proof), give 4 points.

Give a maximum of 3 points for any solution with a wrong answer. Solutions in this category that earn 3 points should be almost entirely correct except for a mistake in a case or two.

Here is a full listing of all 33 possibilities, for reference:

IGABCDHEF	GABICDEFH	GAIBDCEHF
IGABCDEHF	IGABDHCEF	GAIBDCEFH
IGABCDEFH	IGABDCHEF	GABIDHCEF
GIABCDHEF	IGABDCEHF	GABIDCHEF
GIABCDEHF	IGABDCEFH	GABIDCEHF
GIABCDEFH	GIABDHCEF	GABIDCEFH
GAIBCDHEF	GIABDCHEF	GABDIHCEF
GAIBCDEHF	GIABDCEHF	GABDICHEF
GAIBCDEFH	GIABDCEFH	GABDICEHF
GABICDHEF	GAIBDHCEF	GABDICEFH
GABICDEHF	GAIBDCHEF	GABDHICEF

Problem 2/1/24:

The solution to this problem has two parts. One is to find the shape of the dot's path, worth 2 points. The other is to compute the length of each piece, worth 3 points. As usual, a correct solution with a method that does not neatly divide in this way is worth 5 points.

Path shape: Award 2/2 points for solutions which correctly describe the path of the dot. Award 1/2 points if at least one of the 8 arcs of the path is correctly determined.

Path length: Award 3/3 points for solutions which correctly compute the length of all 8 arcs of the path and their sum. Award 2/3 points for a mostly correct computation with minor errors. Solutions which do not correctly compute the path's shape can be awarded at most 1/3 points for this part.

Problem 3/1/24:

5 points for a correct configuration. Award 1 total point for configurations which satisfy all the constraints except one (e.g. some people did not realize the subsets should be distinct). Award 1 total point if at least three squares are correctly filled in. Award 4 total points if the entire configuration is right save for one or two errors.

If a configuration is not fully correct, you should specifically point out at least one error in your comments. A correct proof of uniqueness should be commended in the comments.

Problem 4/1/24:

The solution to this problem has two parts. One is to handle all numbers between 1 and $2^m - 1$, worth 3 points. The other is to show how to shift up by $2^m - 1$, worth 2 points. As usual, a correct solution with a method that does not neatly divide in this way is worth 5 points.

Between 1 and $2^m - 1$: Award 3/3 points for solutions which correctly show all numbers from 1 to $2^m - 1$ are included in the $S_{m,i}$ exactly once. Award 2/3 points for seeing and describing the pattern, perhaps through binary expansions or the exponent of 2 in prime factorizations, but without proving it holds. Award 1/3 points for more cursory ideas, such as noticing that exactly 2^{m-i} numbers of $S_{m,i}$ are less than 2^m.

Shifts by $2^m - 1$: Award 2/2 points for showing $a \in S_{m,i}$, then $a + 2^m - 1 \in S_{m,i}$. Award 1/2 points for noticing this pattern in special cases but not in general.

Solutions that show the sets for specific m and describe why the problem is true in these cases, but fail to make these arguments in general, are worth at most 2 points.

Problem 5/1/24:

The solution to this problem has two parts. One is to find the identity $y_4 - 3y_3 + 3y_2 - y_1 = 0$, worth 2 points. The other is the computations with this identity to finish the problem, worth 3 points. As usual, a correct solution with a method that does not neatly divide in this way is worth 5 points.

Identity $y_4 - 3y_3 + 3y_2 - y_1 = 0$: Award 2/2 points for solutions which show four numbers (y_1, y_2, y_3, y_4) are quadratic *if and only if* $y_4 - 3y_3 + 3y_2 - y_1 = 0$. Do not deduct points if the identity is cited as well-known. Award 1/2 points for solutions which begin doing computations that would lead to this identity but do not finish or make errors. An example of this is a solution that writes $y_1 = a + b + c$, $y_2 = 4a + 2b + c$, $y_3 = 9a + 3b + c$, and $y_4 = 16a + 4b + c$, and then begins doing eliminations on the variables.

Finishing the problem: Award 3/3 points for correctly doing the computations using the above identity to show four rows, three columns quadratic implies the last column is quadratic. Award 2/3 points if minor errors are made. No partial points can be awarded for this section if the identity above was not found.