



USA Mathematical Talent Search  
Round 3 Grading Criteria  
Year 23 — Academic Year 2011–2012  
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**IMPORTANT NOTE:** On **all** problems, the graders have the discretion to deduct 1 additional point for a solution that is poorly written.

**Problem 1/3/23:**

On this problem, give 5 points for a correctly completed grid. **No explanation nor justification is necessary** to receive full credit.

- Deduct 2 points if a row of numbers is missing.
- Deduct 2 points if two numbers are transposed (swapped), and deduct an additional point for each additional transposition.
- Deduct 1 point for a typo.

**Problem 2/3/23:**

- Give 5 points for a correct answer with justification.
- Give 1 point for a correct answer without justification. (This includes simply plugging the sum into a computer.)
- Give 2 points if there is no answer, but the solution correctly splits the sum into parts that are easier to handle.
- Give 3 points if the solution correctly splits the sum into parts, but one of the parts is calculated incorrectly.
- Deduct 1 point for each minor calculation error.

**Problem 3/3/23:**

The solution consists of three parts:

- (2 points) Setting up the correct diagram (in particular, recognizing that the shape of the knot is a regular heptagon with a hole, consisting of seven identical isosceles trapezoids)
- (1 point) Labelling the isosceles trapezoid with the correct distances and angles
- (2 points) Computing the area of the isosceles trapezoid and the strip correctly

If the solution assumed that there were six isosceles trapezoids but not seven, and everything else is correct, then deduct one point.



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Note that the final answer can be expressed in different forms, such as

$$7 \cot \frac{2\pi}{7} + 7 \csc \frac{2\pi}{7}, \quad 7 \tan \frac{3\pi}{14} + 7 \sec \frac{3\pi}{14}, \quad \text{etc.}$$

**Problem 4/3/23:**

Part (a) is worth 2 points. Give 1 point for a diagram that has a minor error, but is otherwise correct. Give 2 points for a correct diagram.

Part (b) is worth 3 points.

- Give 1 point for some progress towards a solution, but which makes incorrect assumptions. (For example, even in an optimal configuration, not every empty cell has to be accessible by Renata.)
- Give 2 points for a rigorous argument that has minor errors.
- Give 3 points for a correct argument.

**Problem 5/3/23:**

The case where  $k$  is even is worth 2 points. Give 1 point for stating the correct answer (i.e. that Xavier has a winning strategy), and 1 point for proving it.

The case where  $k$  is odd is worth 3 points. Give 1 point for a winning strategy for Elise (such as choosing  $e = 1$  and  $f = 9$ ), without justification. Give 2 points for partial work, and give 3 points for a correct argument.