USA Mathematical Talent Search

Round 2 Grading Criteria

Year 23 — Academic Year 2011–2012

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IMPORTANT NOTE: On **all** problems, the graders have the discretion to deduct 1 additional point for a solution that is poorly written.

Problem 1/2/23:

Award 1 point for each correct answer (out of 2 total).

Award the remaining 3 points for clear exposition:

- Give 1 point for recognizing all possible adjacent pairs of numbers.
- Give 1 point for recognizing that 8 and 16 must be placed at the ends.
- Give 1 point for dealing with the “3” or “1” case.

If the submission gives additional erroneous answers, then the solution gets one point total, unless the additional answers are the natural result of a trivial error, such as writing down an erroneous adjacent pair.

Problem 2/2/23:

There are two fundamentally different approaches to this problem. In both cases give 1 point for the answer, 2 points for the algorithm, and 2 points for the exposition.

Mathematical Solutions: Students must provide the correct answer (1 point). Students must have, either explicitly or implicitly, the correct algorithm (2 points). Finally, the student must have a proof that the only way to give an equal mixing is by following the correct algorithm (2 points).

Programming Solutions: Students who write a program must arrive at the correct answer (1 point), meaning an exact numerical answer or an argument that some floating point solution must round to the correct answer. Students must also have a correct and valid algorithm (2 points). Finally, students must explain why their programs work and give the correct solution (2 points).

Problem 3/2/23:

Award 3 points for showing that 3 and all positive multiples of 10 are possible values of b. Award 2 points for showing that no other values are possible.

- A solution that proves all positive multiples of 10 are possible values of b should receive at least 1 point.
- Give at most three points to any solution that breaks the problem into casework, but does not find the answer $b = 3$.
- A solution that proves all positive multiples of 10 are possible and also that 3 is possible should receive at least 3 points.

- A solution that exhausts on a small range of numbers, such as any number less than a million, and finds 3 and multiples of 10 as possible values does not get the 2 points for showing that no other values are possible.
- A solution that finds all solutions, and makes a good attempt at the proof with only minor errors may receive 4 points.
- Give 5 points for a clear, complete, and correct solution.

Giving -2 as an answer loses 2 points.

Problem 4/2/23:

The key points of this problem are:

- Recognizing that the centers of all arcs lie on the perpendicular bisector of $\overline{AB}$.
- Recognizing that the extreme arcs are found by intersecting perpendiculars from A and B with the bisector.
- Proving that the extreme arcs are found by intersecting perpendiculars from A and B with the bisector.
- Computation.

These should be weighted, roughly, 1 point, 1 point, 2 points, and 1 point.

Points should be awarded as follows.

- Give at least 1 point for any solution that recognizes the arcs are centered on the perpendicular bisector.
- Give 2 points for any solution with the correct diagram and an arithmetic error.
- Give 3 points for any solution with the correct diagram and the correct answer.
- Give 3 points for any solution with the correct diagram, a flawed argument that the correct luns is the one given, and an arithmetic error.
- Give 4 points for any solution with the correct value, but a flawed attempt at proving the target luns is maximal.
- Give 4 points for a clear and complete argument with an arithmetic error.
- Give 5 points for a clear and complete argument with the correct solution.

Problem 5/2/23:

One common mistake is to assume that friendship is transitive. This causes best friends to be gathered into cliques of up to four mutual friends and makes the problem much simpler. Award up to 2 points for a successful solution for this interpretation of the problem.

- Award 1 point for finding that a student would have a neighborhood of 28 other students within the 3-meter distance.
- Award 5 points for a correct solution that swaps two students after eliminating the six sets of 28 that could have possible best friends as possible swap locations.
- Award 5 points for a solution that organizes the students so well that it is possible to seat them in the first attempt without creating disruptive pairs. All such solutions involve dividing the students into groups that have at most two friends of each student in the same group, so that friendships could create paths or cycles in the social network but not more complicated subgraphs.
- Deduct 1 point for any gap in logic or weak explanation in an otherwise correct solution.