USA Mathematical Talent Search

Round 1 Grading Criteria

Year 23 — Academic Year 2011–2012

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IMPORTANT NOTE: On **all** problems, the graders have the discretion to deduct 1 additional point for a solution that is poorly written.

Problem 1/1/23:

On this problem give 5 points for a correctly completed grid. **No explanation nor justification is necessary** to receive full credit.

Otherwise, give 4 points to a solution that swaps n with $7-n$ everywhere (a correct solution to the puzzle with reversed arrows). Give 1 point for a partial solution with some numbers correct. Give 0 points to any solution with any numbers misplaced.

Problem 2/1/23:

Showing that $(a,b,c,d,e) = (3,2,1,5,4)$ is the only possible solution is worth 4 points. Actually showing that the solution is valid is the other point. (That is, a solution that establishes $(3,2,1,5,4)$ as the only solution to a single quadratic equation, without verifying that the solution indeed works in all five original equations, is only worth 4 out of 5 points.) Note: If a student states that the solution actually satisfies the five original equations but does not explicitly perform the arithmetic, the solution should still be awarded 5 points.

Algebraically bounding the values (a,b,c,d,e) and performing an exhaustive computer search on the restricted region is a valid 5-point solution. If the proof of the bounds is incorrect or missing, then the solution is worth at most 3 points.

Problem 3/1/23:

The solution to this problem has five pieces. The student must determine that the answer is "Yes," explain a strategy that produces a real coin in all cases, must prove that the strategy works, must prove that the algorithm actually terminates, and must do all of this in a clear and precise manner. These five things are weighted more-or-less equally as below. Award 2 points for the strategy (including the correct answer, "Yes") and 3 points for the proof that the strategy is successful (including termination and clarity).

Award at most 1 point to any student who says "No," and only if the student proves an interesting, nontrivial result. Award at most 2 points for a student with an invalid strategy and only in extremely rare circumstances.

The strategy should be broken down as follows: Give 1/2 for any affirmative strategy containing any nontrivial concepts. Give 2/2 for any strategy that is valid.

The proof that the strategy is correct should be broken down as follows:

- Award at most 1/3 points for justification of an invalid strategy. (Therefore an invalid strategy is worth *at most* 2/5, and only in very exceptional cases.)
- Award 1/3 for an unclear or incomplete atttempt at justifying a valid strategy that does not prove termination, but that does make significant progress.
- Award 2/3 for a clear solution that fails to deal with termination of the algorithm.

- Award 2/3 for an unclear or incomplete justificaton that does make significant progress and argues for termination.
- Award 3/3 for a clear, complete justificaton that also argues that the algorithm will terminate.

Problem 4/1/23:

Part (a) of this problem is worth 3 points and has 2 parts: First a student must show that $\frac{1}{2}$ is an upper bound for all the possible radii of enclosed spheres. Second, the student must show that this value is achievable. Award 1 point for each of these three tasks (for a total of 3 points):

- Give 1 point for mentioning the value of $\frac{1}{2}$.
- Give 1 point for showing (explicitly or implicitly) that there exists a sphere of radius $\frac{1}{2}$ that lies inside P. This requires dealing with all 7 faces.
- Give 1 point for arguing that no larger sphere can be embedded in the polyhedron.

Part (b) of this problem is worth 2 points and has 2 parts: A student must restrict to the line $\overleftrightarrow{XY}$ and then must find the length XY. Award 1 point for a complete argument pinning the solution set to the line $\overleftrightarrow{XY}$ and award the second point for computing the distance from X to Y.

Problem 5/1/23:

A correct and complete solution is worth 5 points. A solution that does not correctly and completely explain an otherwise correct strategy is worth 3 or 4 points, depending on the missing data.

A submission that does not contain a complete strategy is worth at most 2 points. A submission that makes a useful and insightful observation may earn 2 points. A solution that provides some minor insight (such as explaining what the stacks looked like the step before the solution) may earn 1 point. Some examples of good, nontrivial observations are:

- Describing how to win at Tristack Solitaire if one of the stacks has exactly one card.
- Describing how to win at Tristack Solitaire if the sum of the sizes of a pair of stacks is a power of two.
- Describing why a series of legal moves on a pair of stacks with one odd size and one even size, the pair of stacks will eventually repeat and the previous move will give the "inverse" operation.