IMPORTANT NOTE: On **all** problems, the graders have the discretion to deduct 1 additional point for a solution that is poorly written.

Problem 1/4/21:

1 point: Wrong answer, but understood problem

3 points: wrong answer, but generally the right idea, minor error in logic, such as not noticing that circling p^2, pq, and q^2 will cross out all other $p^l q^m$.

4 points: minor arithmetic error.

5 points: complete and correct.

A computer-program solution is OK if it explains the algorithm and covers all possibilites, or explains why it doesn't have to.

Problem 2/4/21:

1 point: Proved that $a^2 + b^2 + c^2 + d^2 = 40$, or found one solution for a, b, c, and d.
2 points: Assumed that a, b, c, and d had to be integers.
3 points: Declared that $a = b = c$ without proof.
4 points: Proved the correct upper bound on d, but did not verify that a solution exists with that value for d. Or used the form of the AM-GM rule that applies only to positive numbers to create the upper bound on d.
5 points: Proved the correct upper bound on d and verified that values for a, b, and c exist that attain that value for d.

Problem 3/4/21:

The problem is worth 3 points for stating and proving the observation about how many turns are necessary for any particular configuration, and then 2 points for the expected value computation.

For determining how many turns are necessary: give 1 point for the correct assertion, 2 points for a plausible argument that is not rigorous, and 3 points for the correct assertion with proof.

For computing the expected value: 1 point for setting up a formula (often a recursive formula) but something short of a corrct final answer, and 2 points for a correct formula just in terms of n. The summation cannot be removed, so any correct formula involving a summation (or an equivalent expression with $+ \cdots +$ in it that is clearly presented) should get the full 2 points for this part. The summation does *not* need to be simplified.

Problem 4/4/21:

5, 4, or 3 points for a mostly correct solution (deducting 1 or 2 points for a minor or major flaw, respectively).

0, 1, or 2 points for an essentially incorrect solution (awarding 1 or 2 points, respectively, for some minor or major progress, respectively).

Correct solutions will almost certainly have a Pigeonhole-type step somewhere, either explicit or implicit (as in the provided solution). Those that don't are very likely incorrect. The most common solution method is to use the intervals

$$\left(0, \frac{1}{8}\right], \left(\frac{1}{8}, \frac{2}{7}\right], \left(\frac{2}{7}, \frac{3}{6}\right], \ldots, \left(\frac{7}{2}, 8\right], (8, \infty)$$

as the "holes"—any two numbers in the same hole will satisfy the inequality.

Problem 5/4/21:

Split into two parts: 3 points for proving that the center is the optimal point, then 2 points for computing the probability of the center point. Students that assert (with no proof or an incorrect proof) that the center is optimal can still receive the 2 points for the computation.

For the proof that the center is optimal: deduct 1 point if the solution does not account for the cases where the white region are not quarter-circles (this occurs if T is close to one of the corners), and deduct 1 or 2 points (depending on the severity) if the student does not address the concept that one can treat the two pairs of opposite corners simultaneously (i.e. maximizing W_A and W_C is simultaneous with maximizing W_B and W_D), if this is a necessary part of their argument. A bogus "symmetry" argument that "proves" the center is best without doing the necessary work is 1 point at most for this part, and in most cases is 0 unless the student does some significant nontrivial work. Drawing the correct picture is not worth any points for this part: they must make some nontrivial progress towards showing that the center is optimal to receive any credit on this part.

For the computation of the center's probability: 1 point for the correct picture or idea, and 1 point for the correct computation. However, award 0 out of 2 points for any answer not strictly between 0 and 1.