IMPORTANT NOTE: On **all** problems, the graders have the discretion to deduct 1 additional point for a solution that is poorly written.

Problem 1/2/21:

1 point for part (a): must be correct with correct explanation. 1 point may also be earned if part (a) is skipped but part (b) is done correctly *and* the student notes explicitly that part (a) follows from part (b).

4 points for part (b): 1 point for noticing that using 1 is a key step, 2 points for a plausible approach but with a major flaw, 3 points for essentially the correct approach but with a minor flaw, 4 points complete and correct.

Many students did it the opposite way as the solution above: they first multiplied and then repeatedly added 1. This is valid but slightly more difficult to make rigorous.

Any flaw in reasoning, rather than sloppiness in rigor, loses at least 2 points. Asserting, without proof, that there exists a smallest solution to $a + n \le bn$ loses 1 point.

Problem 2/2/21:

The correct answer with a reasonable explanation will almost always score 5. This includes a computer program (with explanation) that produces the correct answer.

Give 0/5 for any solution with an answer greater than $2^{16} = 65536$.
Give 1/5 for a reasonable start. Also 1/5 for a solution with no explanation or a computer solution with no justification.
Give 2/5 for some valid progress. Also 2/5 is the most a solution with a highly unreasonable answer can receive (e.g. an answer much less or much greater than 2926).
Give 3/5 for a plausible method with a significant error.
Give 4/5 for a correct method with a minor error (e.g. overlooking a case or incorrect arithmetic).

Problem 3/2/21:

Essentially correct solutions should receive 3, 4, or 5. Give 4 if there is a minor flaw or the argument is poorly written. Give 3 out of 5 if the idea of the solution is essentially correct but there is a significant flaw. The inductive argument need not be explicit: the level of detail shown in the published solution is sufficient.

Essentially incorrect solutions should receive 0, 1, or 2. Give 2 for a solution that makes some nontrivial progress (for example, showing that $7^{2010}|(a^2 + b^2)$). Give 1 for solutions that take a reasonable approach.

Any theorems cited must be cited by name or reference and include a complete statement of the theorem.

Problem 4/2/21:

2 points for part (a). We do not require the most efficient solution—any valid path should receive full credit.

3 points for part (b). Award 1/3 for demonstrating a correct idea (e.g. that Pythagorean triples are significant and/or that only even denominators can be attained); award 2/3 for an essentially correct argument with a *minor* flaw; 3/3 for correct and complete.

Problem 5/2/21:

No points for a solution that claims that $PQ = 2.5$ is the maximum.

Many students used calculus. A student who did the calculus via computer but did not prove that it worked gets at most 3/5. A student who showed the calculus but then used a decimal approximation at the end to establish the inequality gets at most 4/5. A student who used calculus incorrectly gets at most 1/5.

Essentially correct solutions should get 3, 4, or 5: 4/5 for minor flaws and 3/5 for an essentially correct argument but with major flaws. Essentially incorrect solutions should get 0, 1, or 2: 1/5 must make some nontrivial progress and 2/5 for significant progress towards the correct solution.