



# USA Mathematical Talent Search

Round 1 Grading Criteria

Year 21 — Academic Year 2009–2010

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**IMPORTANT NOTE:** On **all** problems, the graders have the discretion to deduct 1 additional point for a solution that is poorly written.

## Problem 1/1/21:

For programming solutions: 5/5 for a correct solution with explanation and justification of code; downgrade to 4/5 (or lower) if essentially correct but inadequate explanation. 1/5 for an incorrect answer accompanied by well-explained code (meaning a detailed explanation of all steps). 1/5 for the correct answer with no explanation (even if the code appears).

Non-programming solutions:

- 1/5: The student found a solution to filling in the circles or otherwise demonstrated that he or she understood the rules for filling in the circles.
- 2/5: Student observes that the lower center circle must be assigned 1 or 8; the student counts many invalid solutions in which he or she put consecutive digits in circles joined by a line; the student constructs one solution, pointing out the number of choices at each step, and then multiplies the number of choices together to get the total number of solutions, not realizing that different choices could lead to a different number of choices in a later step.
- 3/5: The student had a reasonable method for finding all solutions but made errors that either left out solutions or miscounted solutions.
- 4/5: Correct solution, but left out a (minor) piece of the proof.
- 5/5: Correct and justified solution.

## Problem 2/1/21:

For programming solutions: 5/5 for a correct solution with explanation and justification of code. 1/5 for an incorrect answer accompanied by well-explained code (meaning a detailed explanation of all steps). 1/5 for the correct answer with no explanation (even if the code appears).

Non-programming solutions:

- 1/5: Plausible start.
- 2/5: Significant progress towards a solution but with severe shortcomings; solved 5-digit case but neglected 6-digit case.
- 3/5: Bad treatment of 6-digit case (especially invalid nonexistence argument). Max value for an otherwise perfect solution which does not list and eliminate the four “carrying” cases. Max value for a solution with extraneous answers.



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- 4/5: Correct answer with some cases overlooked (e.g.  $11111 = 1 \cdot 11111$ )
- 5/5: Correct and complete answer.

### Problem 3/1/21:

On this problem, break up credit as 2 points for the correct answer and 3 points for the explanation.

For programming solutions: 5/5 for a correct solution with explanation and justification of code. 1/5 for an incorrect answer accompanied by well-explained code (meaning a detailed explanation of all steps). 2/5 for the correct answer with no explanation (even if the code appears).

Nonprogramming solutions:

On the explanation (which is 3 of the 5 total points):

- 1/3 for some nontrivial progress.
- 2/3 for an argument with minor flaws (This is the maximum value for any explanation which does not use that 1489 is coprime to 689).
- 3/3 for correct and complete explanation.

### Problem 4/1/21:

This problem has two parts. First one needs to prove that no solutions beyond 40 and 60 are allowable and second one must prove that 40 and 60 are both attainable.

Solutions that only show one solution (40 or 60) should receive at most 2 out of 5 points.

Finding that 40 and 60 are the only viable solutions is worth a total of 3 points:

- 1/3 for minor progress.
- 2/3 for a complete explanation which overlooks one solution or a flawed argument which explicitly eliminates all but 40 and 60 as possibilities.
- 3/3 for a complete and correct solution.

Showing that 40 and 60 are indeed constructible is worth 2 points:

- 1/2 for understanding that this is a necessary step and making any progress toward showing it.
- 2/2 for a complete and correct solution (allowing terse arguments for the trivial angles).



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**Problem 5/1/21:**

Part (a) is worth 2/2 and part (b) is worth 3/3.

Part (a): 1 point for the correct answer and 1 point for a correct, formal proof. Numerical solutions **do not count** as proof.

Part (b):

- 1/3 for any nontrivial progress. This is the maximum value on part (b) for any solution which does not somehow use the necessary fact that  $r < \frac{1}{2}$ .
- 2/3 for a proof which is mostly correct but with minor flaws.
- 3/3 for a correct and complete solution.