USA Mathematical Talent Search

Round 4 Grading Criteria

Year 20 — Academic Year 2008–2009

www.usamts.org

IMPORTANT NOTE: On **all** problems, the graders have the discretion to deduct 1 additional point for a solution that is poorly written.

Note: Situations not explicitly covered by this rubric are graded on the general principle of how much progress was made on the solution, using the cases listed in this rubric as guidelines.

Problem 1/4/20:

Note: solution must show all work to receive full 5 points.

4 points if the work was correct but the solution did not plug x (the ratio between concecutive terms) back into the formula for a_5 in order to answer the actual question of the problem.

2 points for correct method but obtained the wrong value for x.

2 points for solving by computer program without proving that the computer program will find all possible values for a_5.

Problem 2/4/20:

3/5 could be awarded for solving either half alone, either proving $k \leq 8$ impossible or $k = 9$ possible.

Solution with $k = 10$, incorrectly assuming that mutual friends and mutual strangers must be disjoint: 3/5.

One method is to start with a fully connected graph with 5 people, and adding people one by one with some sort of "greedy" algorithm. This is scores: 2/5 if the algorithm gives $k = 10$, 3/5 if it gives $k = 9$, 3/5 if $k = 10$ is incorrectly proved minimal, and 5/5 if $k = 9$ is correctly proved minimal since that is a correct solution.

Single arrangement for $k \geq 10$, no minimality argument: 1/5.

Single arrangement for $k = 9$, no minimality argument: 3/5.

Incorrect arrangement for $k = 9$, with minimality argument: 4/5 if error is trivial (checked but one edge missing or miscounted), otherwise 3/5.

Problem 3/4/20:

Setting up the correct transition probabilities for a large number of states: 2/5.

Using this to build a correct Monte Carlo model with a "good" answer: 3/5.

Using this to build a computer algebra solution (if correct): 5/5.

Setting up a correct set of equations in two or three variables, wrong answer with no supporting work: 3/5.

Setting up an incorrect set of equations in two or three variables by a correct method but with a misunderstanding of the probability: 3/5.

Setting up an incorrect set of equations in two or three variables with a minor error (number copied wrong): 4/5.

Setting up a correct set of equations, right answer: 5/5.

Argument (usually giving the incorrect final answer 5/6) based on the probability of hitting $y = 0$ before $y = 6$ after going horizontally on $y = 2$ being 2/3, or after being at $y = 4$ being 1/3, either based on expected values or an infinite series: 2/5.

Problem 4/4/20:

Most students found the correct solution that $f(x) = x$ for all nonnegative integers x. A few had the false second solution $f(x) = -x$. Thus, the problem was graded on the quality of the proof that $f(x) = x$ is the only solution. We did not require that they verify that the solution is valid, because that is quite obvious.

The vast majority of students used induction to show that $f(m+n) = f(m)+f(n)$ meant that $f(m) = mf(1)$. However, a few students quoted the Cauchy Functional Equation or Jensen's Theorem. In general, we gave full credit for such answers.

Points lost for errors:

Allowed $f(x) = -x$ as a solution: -1 point

Went directly from $f(2f(0)) = 0$ to $f(0) = 0$ without proof: -2 points.

Went directly from $f(f(n)) = n$ to $f(n) = n$ without proof: -2 points.

Assumed without proof that f was invertible: -3 points.

Assumed at the beginning that f was a polynomial function: -4 points.

Problem 5/4/20:

Most correct final answers will get 5/5. But point(s) may still be deducted for a poorly written solution and/or gaps in the proof.

For incorrect answers: deduct 1 if it's only an algebra mistake, but increase this deduction to 2 if the mistake produces an answer that the student should have recognized as wildly incorrect.

Deduct 2 (in general) if the student makes a geometric assumption that is not obvious and was not proved.