



USA Mathematical Talent Search

Round 3 Grading Criteria

Year 20 — Academic Year 2008–2009

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IMPORTANT NOTE: On **all** problems, the graders have the discretion to deduct 1 additional point for a solution that is poorly written.

Note: Situations not explicitly covered by this rubric are graded on the general principle of how much progress was made on the solution, using the cases listed in this rubric as guidelines.

Problem 1/3/20:

1 point for any real work.

3 points for showing that one sum of digits is 17 and the sum is 28.

4 points for finding the 11 ways to divide the digits to get sums of 17 and 28.

Problem 2/3/20:

2 points for a solution that shows that a player can control the number of stones modulo 7, 10, or some other period but does not offer a use for this control. Increase this to 3 points if the student uses this control for some erroneous endgame strategy that would let the player win.

3 points if the solution describes the 11-long period of winning and losing positions without proving the period.

4 points for missing some rigor, such as forgetting to describe the endgame.

Problem 3/3/20:

2 points for showing that $(a, b)(a, c)(b, c)(a, b, c) = abc$.

3 points for the correct 3-prime case, but not eliminating the 2-prime or 4-prime cases.

3 points for correct solution but relied on the unproven assumption that $a + b + c$ is minimized by minimizing the prime factors.



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Problem 4/3/20:

1 point if tried to determine what properties lead to a unique D but never reached the right conclusion.

2 points if determined that the uniqueness of D meant that it was on the perpendicular bisector of \overline{BC} .

3 points for the equation of the correct ellipse or a foci-based description of the ellipse with no proof that the locus is an ellipse.

The solution can get full credit even if the degenerate points at the vertices of the ellipse are included in the locus. If the participant notes that those points must be removed, we give +1 point on any partial credit for some other error.

Problem 5/3/20:

1 point for showing $0 < a < 2$ and nothing else.

1–2 points for demonstrating the $a \leq 1$ case (to get the 2 points generally requires some non-trivial observation beyond just showing the $a \leq 1$ case).

3 points for proving it for just rational a .

3 points for some nontrivial progress on the $a > 1$ case.

4 points for a correct proof that lacks rigor.

5 points for a mostly correct solution with minor flaws.