IMPORTANT NOTE: On **all** problems, the graders have the discretion to deduct 1 additional point for a solution that is poorly written.

Problem 1/2/20:

4 out of 5 for a *minor* gap or a very poorly written (but correct) solution.

3 out of 5 if missing a case, but otherwise correct.

3 out of 5 if neglecting to subtract the positions with three O's in a row. (This would yield an answer of 480.)

2 out of 5 for a "worse" wrong answer (for example, only counting the X's and not paying attention to the O's at all). 1 out of 5 for a semi-reasonable start.

Computer solutions can receive 5 out of 5, provided that the algorithm is explained and justified. Otherwise, 1 out of 5 for a correct computer answer without justification.

Problem 2/2/20:

Break up as 2 points for setting up the correct diagram, and 3 points for computing the answer.

"Setting up the diagram" must prove that the trapezoid is isosceles and that the center circle's point lies where is claimed. Deduct 1 point is these are not done. No points (out of 2) at all if the diagram is asserted without justification.

The algebra is worth the remaining 3 points. Students must show all the significant steps. Deduct a point if, for example $\sqrt{1+(2\sqrt{3}-h)^2}+2=\sqrt{9+h^2}$ magically becomes $h=3/\sqrt{2}$ without any intervening work.

Many students will set up the algebra differently. One common solution uses

$$\sqrt{(r+1)^2-1}+\sqrt{(r+3)^2-9}=2\sqrt{3},$$

where r is the radius. (This is actually probably simpler that the solution above.)

Minus 1 if students do everything right but their final answer is not the radius.

Unsimplified final answers are OK to a certain degree. In particular, no penalty for leaving a radical in a denominator.

Problem 3/2/20:

Overall, 2 points for proving that $p(x)$ must be linear (or constant), 2 points for the correct answers, and 1 point for excluding the $x/4+1/2$ solution.

We're willing to be a bit lenient on the "proof" that $p(x)$ must be linear. Basically, give 1 point for showing that $p(a_n) < a_n$ for all n, and the other point for the "conclusion" that this means that p must be linear. We're not demanding a tightly rigorous argument for this, just something plausible that shows that the student understands what's going on.

For each correct polynomial, students must show that there is indeed a corresponding sequence. They will need to prove that the sequences work in more detail than the solution above.

Some students saw "by inspection" that $p(x) = x/2$ works with the sequence $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$. If they prove this works, give 2 out of 5. (Most students who do this will not get the $x - 1$ solution.) If they find the two solutions but don't prove that no others work, give 3 out of 5.

Problem 4/2/20:

2 points for Lemma 1 (from the published solution, or its equivalent), 2 points for Lemma 2 (or its equivalent), then 1 point for the conclusion. (Of course, students might not phrase it exactly this way, but these are the two key steps to solving the problem.) 0 points for either of these parts if there is no proof (i.e. an assertion without proof that the three numbers must be powers of a prime gets 0 out of 2 points for that part).

For the Lemma 2 part: some students quote "Catalan's Conjecture" that the only two nontrivial consecutive powers of primes are 8 and 9. This is acceptable (though a comment that this is overpowering the problem might be in order; also, it should be properly cited as "Mihăilescu's Theorem" and not as a "conjecture").

Minus 1 if the final answer is not what is asked for (for example, giving N instead of k, or giving $k = 14$ instead of $k = 13$).

Problem 5/2/20:

Roughly:
2 points for setting up a summation for the total area (either with one vertex fixed, as in the published solution, or for the total area of all triangles)
2 points for the algebraic work to get the total area in terms of the required cotangent
1 point for finishing by computing the expected value

In the algebra, deduct 1 point for a key step performed via Mathematica (or other computer/calculator program) without verification.

OK if the answer is unsimplified if it's clearly correct.

Use judgment as to whether trig identites, etc., are "obvious" enough to be used without citation. (Probably most are, though, unless really obscure.)

Students citing the published paper (where this problem appeared) should get 5 out of 5 if properly cited.