USA Mathematical Talent Search

Round 1 Grading Criteria

Year 20 — Academic Year 2008–2009

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IMPORTANT NOTE: On **all** problems, the graders have the discretion to deduct 1 additional point for a solution that is poorly written.

Problem 1/1/20:

This problem is harder than it looks. We're not requiring rigor: an accurate methodical listing of the cases with the correct total is sufficient to get 5/5.

4 points for a total that is *slightly* off because a case was missing or overcounted.

3 points for a total that is "reasonable" (i.e. "near" 22) with a reasonable method that missed or overcounted case(s). Also 3 points for an answer that omits the "white cube in the center" case.

1 or 2 points for some progress and/or solving a simplified version of the problem (e.g. in 2 dimensions)

0 or 1 points for a gross misinterpretation or for very little progress.

Problem 2/1/20:

2 points for the answers (1 point each), and 3 points for the proof.

On the proof (out of 3 points): give 1/3 for some sort of recognition that there is something to prove besides just listing the two answers, 2/3 for a partial proof with some minor flaws, 3/3 for a complete proof.

Computer solutions get 2/5 unless they prove that no $n > 31$ can work.

The most common reason for getting only 4 points overall was that some students got the problem down to a sum of distinct reciprocals $1/a + 1/b + 1/c > 1$, and said that $1/2+1/3+1/4$ and $1/2+1/3+1/5$ were the only sums greater than one without any further explanation. We took one point off for not explaining how they eliminated the other cases.

The most common reason for not getting the correct answers was making the false assumption that since x, y, and z were all factors of $n-1$, then the product xyz was also a factor of $n-1$. Some students wrote a rigorous proof that under those conditions no solutions were possible. We gave them 1 point out of 5 for that proof.

Problem 3/1/20:

Break up points as: 1 point for the correct answer, 1 point for an example establishing the answer, and 3 points for the proof. Many students have the correct answer but with an incorrect explanation.

A wrong answer might get up to 2 points if there is a semi-reasonable method attempted, or (in rare cases) 3 points if it is very close to the correct solution.

Solutions that (correctly) establish the bound of 300 but don't show that it is attainable get 4/5.

Asserting that $c = \pm 100$ without proof but everything else correct: 4 points.

$|a+b+c| \leq 100$ without any other correct work (for example, asserting $|a|+|b|+|c| \leq 100$): 0 points. Increase that to 1 point if anything else is given, such as $|c| \leq 100$ or $|a-b+c| \leq 100$.

Asserting (without proof) that the values of -100 and $+100$ must both occur in the interval $[-1, 1]$: usually 3 points, rarely could be 4 points with good additional work.

Sum of 300 with wrong polynomial (usually $100x^2 + 100x - 100$): treat as wrong answer; 4 points possible if the right polynomial is given as well.

Problem 4/1/20:

5/5 for a completely correct solution, 4/5 for a solution method that makes a minor algebra error but it otherwise correct (but which leads to the wrong answer), except reduce this to 3/5 if the final answer is clearly incorrect (i.e. an absurdly high or low number).

No points for overly simplistic wrong answers (such as $\sqrt{22}$, $\sqrt{22} + \sqrt{34}$, etc.).

Problem 5/1/20:

Give 1/5 if they show that x^2 is groovy and nothing else.

Many students correctly show that $x^r = \sqrt{a} + \sqrt{a+1}$ for some a, but fail to show that a is an integer. Give 2/5 for this. Similar to this is students who show that grooviness is closed under multiplication (it's not); generally give 2/5 for this.

Give 3/5 or 4/5 for a solution that uses a correct method but which has flaws. (4/5 if the flaw is pretty minor, 3/5 otherwise.)