



USA Mathematical Talent Search

Round 2 Problems

Year 38 — Academic Year 2026-2027

www.usamts.org

Important information:

1. **You must show your work and prove your answers on all problems.** If you just send a numerical answer with no proof for a problem other than Problem 1, you will get no more than 1 point.
2. Put your name, username, and USAMTS ID# on **every page you submit.**
3. No single page should contain solutions to more than one problem. Every solution you submit should begin on a new page.
4. Submit your solutions by **November 30, 2026** via one (and only one!) of the methods below:
 - (a) Web: Log on to www.usamts.org to upload a PDF file containing your solutions. (No other file type will be accepted.)
Deadline: 10 PM Eastern / 7 PM Pacific on November 30, 2026.
 - (b) Mail: USAMTS
90 Broad Street
Suite 902
New York, NY 10004
Deadline: Solutions must be postmarked on or before November 30, 2026.
5. Confirm that your email address in your USAMTS Profile is correct. You can do so by logging on to www.usamts.org and visiting the “Account” page.
6. Round 2 results will be posted at www.usamts.org when available. To see your results, log on to the USAMTS website, then go to “My Scores”. You will also receive an email when your scores and comments are available (provided that you did item #5 above).

**These are only part of the complete rules.
Please read the entire rules at www.usamts.org.**



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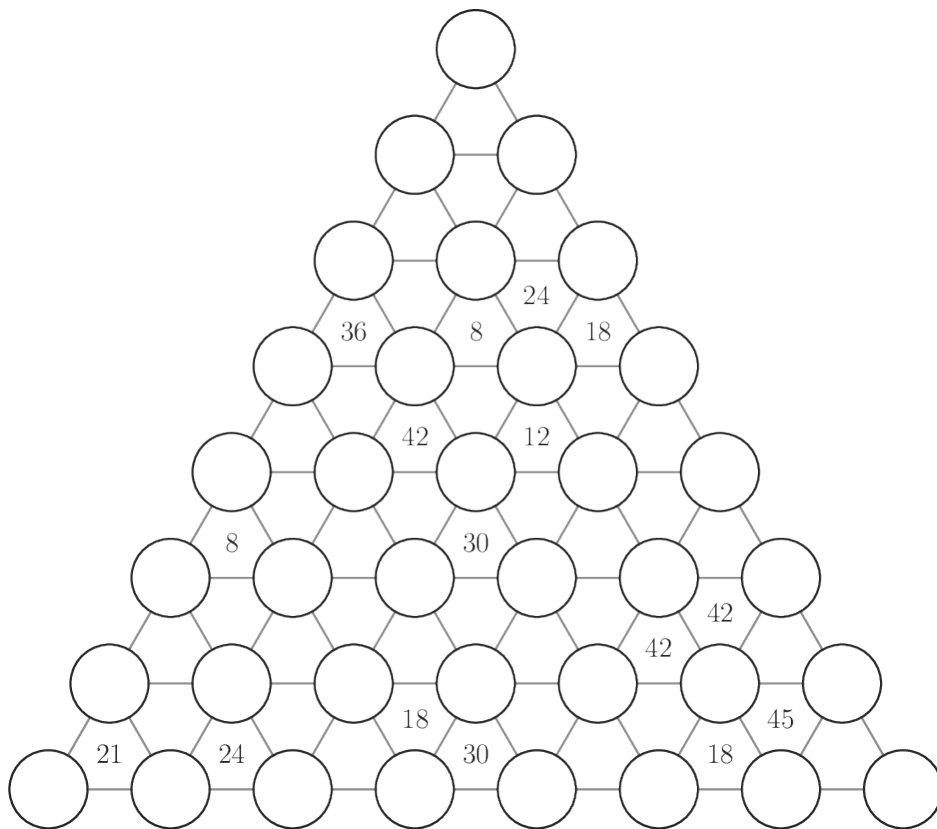
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Each problem is worth 5 points.

1/2/38. Fill in each circle with a digit from 1 to 9. No number appears twice in the same row or diagonal. Numbers within triangles represent the LCM of the three surrounding numbers.

There is a unique solution, but you do not need to prove that your answer is the only one possible. You merely need to find an answer that satisfies the conditions of the problem. (Note: In any other USAMTS problem, you need to provide a full proof. Only in this problem is an answer without justification acceptable.)





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2/2/38. Let S be a string of A 's and B 's. You are allowed to perform the following moves:

- You can change the last letter of S (from an A to a B , or a B to an A).
- Suppose that S ends with a substring of the form $XABBB \dots B$. (In other words, S ends with a letter X , followed by a single A , followed by some number of B 's. The number of B 's can be zero.) Then you can change the letter X (from an A to a B , or a B to an A).

For example, you can change $AABB$ to $AABA$ or $BABB$. Also, you can change $BBAAA$ to $BBAAB$ or $BBABA$.

Find, with proof, the smallest number of moves needed to change $AAA \dots A$ (with 10 A 's) to $BBB \dots B$ (with 10 B 's).

3/2/38. Given a positive integer N , write its factoradic representation with the $0!$ place omitted:

$$N = a_m m! + a_{m-1}(m-1)! + \dots + a_2 2! + a_1 1!,$$

where the a_i are integers such that $0 \leq a_i \leq i$ and $a_m \neq 0$. Define $f(N) = \overline{a_m a_{m-1} \dots a_1}$. In other words, $f(N)$ is the integer obtained by first concatenating all the factoradic digits in the order they appear from left to right, $a_m a_{m-1} \dots a_1$, and then interpreting the resulting string as a decimal integer. For example, $f(2) = 10$, $f(3) = 11$, $f(5) = 21$ and $f(7) = 101$.

Prove that there are infinitely many primes p for which $f(p)$ is not a perfect square.

4/2/38. Let ω be a circle, and let P be a point in the plane. Find the set of points Q that have the following property: There exist points A and B on the circle ω such that $\angle APB = 90^\circ$, and the foot of the altitude from P to line AB is Q .

5/2/38. For an integer $n \geq 2$, let

$$A_n = \left(\underbrace{0.999 \dots 9}_{n \text{ digits}} \underbrace{000 \dots 01}_{n \text{ digits}} \right) \cdot \left(\underbrace{0.999 \dots 9}_{n^2 \text{ digits}} \underbrace{000 \dots 01}_{n^2 \text{ digits}} \right) \cdot \left(\underbrace{0.999 \dots 9}_{n^3 \text{ digits}} \underbrace{000 \dots 01}_{n^3 \text{ digits}} \right) \cdot \dots$$

Prove that A_2 and A_3 are rational, but A_n is irrational for all $n \geq 4$.

Problems by Aidan Deshong and USAMTS Staff.

Round 2 Solutions must be submitted by **November 30, 2026**.

Please visit <https://www.usamts.org> for details about solution submission.

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