



USA Mathematical Talent Search

Round 2 Problems

Year 37 — Academic Year 2025-2026

www.usamts.org

Important information:

1. **You must show your work and prove your answers on all problems.** If you just send a numerical answer with no proof for a problem other than Problem 1, you will get no more than 1 point.
2. Put your name, username, and USAMTS ID# on **every page you submit.**
3. No single page should contain solutions to more than one problem. Every solution you submit should begin on a new page.
4. Submit your solutions by **December 1, 2025** via one (and only one!) of the methods below:
 - (a) Web: Log on to www.usamts.org to upload a PDF file containing your solutions. (No other file type will be accepted.)
Deadline: 10 PM Eastern / 7 PM Pacific on December 1, 2025.
 - (b) Mail: USAMTS
90 Broad Street
Suite 902
New York, NY 10004
Deadline: Solutions must be postmarked on or before December 1, 2025.
5. Confirm that your email address in your USAMTS Profile is correct. You can do so by logging on to www.usamts.org and visiting the “Account” page.
6. Round 2 results will be posted at www.usamts.org when available. To see your results, log on to the USAMTS website, then go to “My Scores”. You will also receive an email when your scores and comments are available (provided that you did item #5 above).

**These are only part of the complete rules.
Please read the entire rules at www.usamts.org.**



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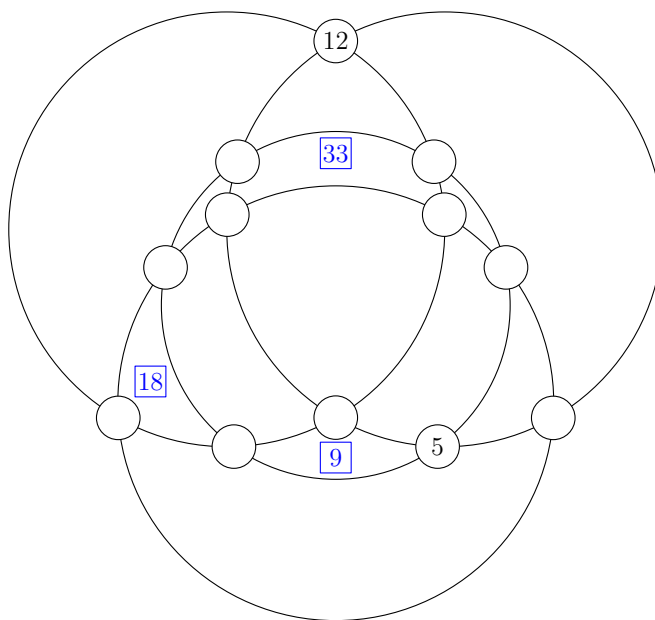
Each problem is worth 5 points.

1/2/37. Place the whole numbers 1 – 12 in the smaller circles appearing at the intersection points of the four larger circles below so that:

- Each whole number 1 – 12 appears exactly once.
- The sum of the 6 numbers around any of the four larger circles is some constant C .
- If two intersection points are adjacent (meaning they are connected by an arc which does not contain other intersection points), then the numbers placed at those vertices are non-consecutive.
- In some regions of the figure there are squares that contain numbers. The sum of the numbers placed in the smaller circles around such regions must be equal to the value inside the square.

The positions of the numbers 5 and 12 have been given for you.

There is a unique solution, but you do not need to prove that your answer is the only one possible. You merely need to find an answer that satisfies the conditions of the problem. (Note: In any other USAMTS problem, you need to provide a full proof. Only in this problem is an answer without justification acceptable.)





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2/2/37. We call a positive integer $n > 4$ *amazing* if, for any list ℓ consisting of $n - 4$ copies of the number 1 and two copies of the number 2, it is always possible to divide ℓ into *at least two* proper contiguous sub-lists which have equal sum.

For example, $n = 6$ is amazing, since we have the following lists with divisions indicated by vertical bars.

- $(1, 1 \mid 2 \mid 2)$
- $(1, 2 \mid 1, 2)$
- $(1, 2 \mid 2, 1)$
- $(2, 1 \mid 1, 2)$
- $(2, 1 \mid 2, 1)$
- $(2 \mid 2 \mid 1, 1)$

Find, with proof, all amazing positive integers.

3/2/37. Let $\mathbb{Z}^+$ denote the set of positive integers. Determine, with proof, whether there exist functions $f, g : \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ such that

$$f(g(g(x))) = 3x,$$

$$g(f(g(x))) = 5x,$$

and

$$g(g(f(x))) = 7x$$

for all $x \in \mathbb{Z}^+$.

4/2/37. Let $ABCDEFGH$ be a regular heptagon with side length 1. Let P be the point on $\overline{EF}$ such that $\angle PAB = 90^\circ$. Compute lengths PC and PD .



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5/2/37. In a round-robin tournament with 1000 teams, each team plays one game against each other team, and each game either results in one of the two teams winning, or a draw. Show that at least one of the following statements must be true:

- There exists a team that draws against at least 10 other teams.
- There exists a group of exactly 10 teams that can be numbered from 1 to 10 such that for all $1 \leq i < 10$, team i won against team $i + 1$, and team 10 won against team 1.
- There exists a group of exactly 10 teams that can be numbered from 1 to 10 such that if two teams are numbered i and j with $i < j$, then team i won against team j .

Problems by Tanny Libman and USAMTS Staff.

Round 2 Solutions must be submitted by **December 1, 2025**.

Please visit <https://www.usamts.org> for details about solution submission.

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