USA Mathematical Talent Search

Round 1 Problems

Year 37 — Academic Year 2025-2026

www.usamts.org

Important information:

1. **You must show your work and prove your answers on all problems.** If you just send a numerical answer with no proof for a problem other than Problem 1, you will get no more than 1 point.

2. Put your name, username, and USAMTS ID# on **every page you submit**.

3. No single page should contain solutions to more than one problem. Every solution you submit should begin on a new page.

4. Submit your solutions by **October 7, 2025** via one (and only one!) of the methods below:

   (a) Web: Log on to www.usamts.org to upload a PDF file containing your solutions. (No other file type will be accepted.)
   Deadline: 10 PM Eastern / 7 PM Pacific on October 7, 2025.

   (b) Mail: USAMTS
   55 Exchange Place
   Suite 503
   New York, NY 10005
   Deadline: Solutions must be postmarked on or before October 7, 2025.

5. Confirm that your email address in your USAMTS Profile is correct. You can do so by logging on to www.usamts.org and visiting the "Account" page.

6. Round 1 results will be posted at www.usamts.org when available. To see your results, log on to the USAMTS website, then go to "My Scores". You will also receive an email when your scores and comments are available (provided that you did item #5 above).

These are only part of the complete rules.
Please read the entire rules at www.usamts.org.

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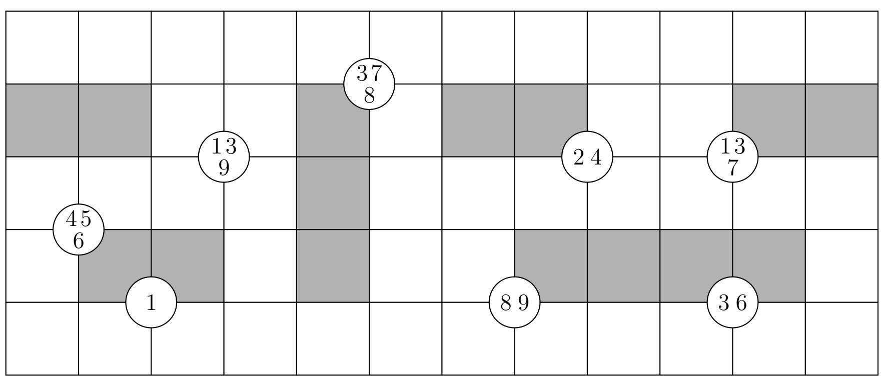
Each problem is worth 5 points.

1/1/37. Place a nonzero digit into some of the white cells of the grid. Shaded cells must remain blank. No digit can repeat in a row or column. In each row, the sum of the digits must be equal to some fixed value R (which you find during the solution). Similarly, in each column, the sum of the digits must be equal to some fixed value C. Circles in the grid give all the digits in the cells that touch the circle. (Including repeats; if two cells touching a circle have the same digit, the circle must contain that digit twice.) Some of the digits you place may not be adjacent to one of the circles, but every digit in the circles must be used in an adjacent white cell.

There is a unique solution, but you do not need to prove that your answer is the only one possible. You merely need to find an answer that satisfies the conditions of the problem. (Note: In any other USAMTS problem, you need to provide a full proof. Only in this problem is an answer without justification acceptable.)



2/1/37. When Nia was learning about decimals, her teacher asked her to add two terminating decimals with a positive integer part. Then, Nia's teacher asked her to multiply those same two decimals. To Nia's surprise, the (correct) result of both computations was the same!

Both of the two numbers Nia's teacher gave her were non-whole positive numbers. When written as decimals without trailing zeroes, both of these numbers also had the same number of digits after the decimal point, and did not contain the digit 0. Find all possible pairs of numbers Nia's teacher could have given her.

3/1/37. Let (a, b) be a pair of rational numbers. Every minute, we are allowed to modify the pair in one of the following ways:

i. Replace (a, b) with $(a + 1, b + 1)$

ii. If $a \neq 0$ and $b \neq 0$, replace (a, b) with $\left(\frac{1}{a}, \frac{1}{b}\right)$

iii. Replace a with $-a$

iv. Replace b with $-b$

(a) Suppose we start with the pair $(0, 0)$. Is it possible to modify this pair to eventually equal $\left(2025, \frac{1}{2025}\right)$?

(b) Suppose we start with the pair $(0, 1)$. Is it possible to modify this pair to eventually equal $\left(2025, \frac{1}{2025}\right)$?

4/1/37. Let S be a finite set of points in the plane such that no three points in S are collinear. Suppose that there are two triangles whose vertices are six distinct points in S, such that their intersection is a hexagon with no points of S in its interior. Prove that there is a convex hexagon with vertices in S that has no points of S in its interior.

5/1/37. Let n be a positive integer. Call a coloring of a rectangular grid k-*good* if

- Each cell of the grid is colored with one of n colors,
- There are the same number of cells of each color, and
- Every row and column has at least k cells of the same color. (The color with at least k cells can vary across rows and columns, e.g., if two of the colors are red and blue, then it's possible for the first row to have at least k red cells, while the second row has at least k blue cells.)

For all n, find the largest positive integer k (which may or may not depend on n) such that there exists a k-good coloring of an $n^2 \times n^2$ grid.

Problems by Tanny Libman and USAMTS Staff.
Round 1 Solutions must be submitted by **October 7, 2025**.
Please visit **https://www.usamts.org** for details about solution submission.