



# USA Mathematical Talent Search

Round 1 Problems

Year 32 — Academic Year 2020-2021

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## Important information:

1. **You must show your work and prove your answers on all problems.** If you just send a numerical answer with no proof for a problem other than Problem 1, you will get no more than 1 point.
2. Put your name, username, and USAMTS ID# on **every page you submit.**
3. No single page should contain solutions to more than one problem. Every solution you submit should begin on a new page.
4. Submit your solutions by **October 19, 2020** via one (and only one!) of the methods below:
  - (a) Web: Log on to [www.usamts.org](http://www.usamts.org) to upload a PDF file containing your solutions. (No other file type will be accepted.)  
**Deadline: 10 PM Eastern / 7 PM Pacific on October 19, 2020.**
  - (b) Mail: USAMTS  
55 Exchange Place  
Suite 603  
New York, NY 10005  
**Deadline: Solutions must be postmarked on or before October 19, 2020.**
5. Once you send in your solutions, that submission is final. You cannot resubmit solutions.
6. Confirm that your email address in your USAMTS Profile is correct. You can do so by logging on to [www.usamts.org](http://www.usamts.org) and visiting the “My USAMTS” pages.
7. Round 1 results will be posted at [www.usamts.org](http://www.usamts.org) when available. To see your results, log on to the USAMTS website, then go to “My USAMTS”. You will also receive an email when your scores and comments are available (provided that you did item #6 above).

**These are only part of the complete rules.  
Please read the entire rules on [www.usamts.org](http://www.usamts.org).**



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Each problem is worth 5 points.

1/1/32. Fill in each empty cell of the grid with a digit from 1 to 8 so that every row and every column contains each of these digits exactly once. Some diagonally adjacent cells have been joined together. For these pairs of joined cells, the same number must be written in both.

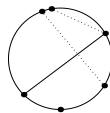
1							
2	3						
	4	5					
		6		1			
			7		1		
					3	4	
						2	8
							5

There is a unique solution, but you do not need to prove that your answer is the only one possible. You merely need to find an answer that satisfies the constraints above. (Note: In any other USAMTS problem, you need to provide a full proof. Only in this problem is an answer without justification acceptable.)

2/1/32. Is it possible to fill in a  $2020 \times 2020$  grid with the integers from 1 to 4,080,400 so that the sum of each row is 1 greater than the previous row?

3/1/32. The bisectors of the internal angles of parallelogram  $ABCD$  determine a quadrilateral with the same area as  $ABCD$ . Given that  $AB > BC$ , compute, with proof, the ratio  $AB/BC$ .

4/1/32. Two beasts, Rosencrans and Gildenstern, play a game. They have a circle with  $n$  points ( $n \geq 5$ ) on it. On their turn, each beast (starting with Rosencrans) draws a chord between a pair of points in such a way that **any two** chords have a shared point. (The chords either intersect or have a common endpoint.) For example, two potential legal moves for the second player are drawn below with dotted lines.



The game ends when a player cannot draw a chord. The last beast to draw a chord wins. For which  $n$  does Rosencrans win?

5/1/32. Find all pairs of rational numbers  $(a, b)$  such that  $0 < a < b$  and  $a^a = b^b$ .

*Problems by Luke Martin, Elena Nefedova, Remus Nicoara, Michael Tang, and USAMTS Staff.*

Round 1 Solutions must be submitted by **October 19, 2020**.

Please visit <http://www.usamts.org> for details about solution submission.

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