USA Mathematical Talent Search

PROBLEMS

Round 2 - Year 13 - Academic Year 2001-2002

1/2/13. How many positive five-digit integers are there consisting of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, in which one digit appears once and two digits appear twice? For example, 41174 is one such number, while 75355 is not.

2/2/13. Determine, with proof, the positive integer whose square is exactly equal to the number

$$1 + \sum_{i=1}^{2001} (4i - 2)^3 .$$

[A computer solution will be worth at most 1 point.]

3/2/13. Factor the expression

$$30(a^2 + b^2 + c^2 + d^2) + 68ab - 75ac - 156ad - 61bc - 100bd + 87cd .$$

4/2/13. Let $X = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9)$ be a 9-long vector of integers. Determine X if the following seven vectors were all obtained from X by deleting three of its components:

$$Y_1 = (0, 0, 0, 1, 0, 1),\ Y_2 = (0, 0, 1, 1, 1, 0),\ Y_3 = (0, 1, 0, 1, 0, 1),\ Y_4 = (1, 0, 0, 0, 1, 1),$$
$$Y_5 = (1, 0, 1, 1, 1, 1),\ Y_6 = (1, 1, 1, 1, 0, 1),\ Y_7 = (1, 1, 0, 1, 1, 0).$$

5/2/13. Let R and S be points on the sides $\overline{BC}$ and $\overline{AC}$, respectively, of ΔABC, and let P be the intersection of $\overline{AR}$ and $\overline{BS}$. Determine the area of ΔABC if the areas of ΔAPS, ΔAPB, and ΔBPR are 5, 6, and 7, respectively.

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Complete, well-written solutions to at least two of the problems above, accompanied by a **Cover Sheet,** should be sent to the address listed on the USAMTS web site

`http://www.nsa.gov/programs/mepp/usamts.html`

and **postmarked no later than 25 November 2001**. Each participant is expected to develop solutions without help from others.