# U S A Mathematical Talent Search 

## PROBLEMS

## Round 2-Year 11 - Academic Year 1999-2000

$\mathbf{1 / 2 / 1 1}$. The number N consists of 1999 digits such that if each pair of consecutive digits in N were viewed as a two-digit number, then that number would either be a multiple of 17 or a multiple of 23 . The sum of the digits of N is 9599 . Determine the rightmost ten digits of N .
$\mathbf{2 / 2} \mathbf{1 1}$. Let $C$ be the set of non-negative integers which can be expressed as $1999 s+2000 t$, where $s$ and $t$ are also non-negative integers.
(a) Show that $3,994,001$ is not in $C$.
(b) Show that if $0 \leq n \leq 3,994,001$ and $n$ is an integer not in $C$, then $3,994,001-n$ is in $C$.
$3 / 2 / 11$. The figure on the right shows the map of Squareville, where each city block is of the same length. Two friends, Alexandra and Brianna, live at the corners marked by A and B, respectively. They start walking toward each other's house, leaving at the same time, walking with the same speed, and independently choosing a path to the other's house with uniform distribution out of all possible mini-mum-distance paths [that is, all minimum-distance paths are equally likely]. What is the probability they will meet?


4/2/11. In $\triangle P Q R, \mathrm{PQ}=8, \mathrm{QR}=13$, and $\mathrm{RP}=15$. Prove that there is a point S on line segment $\overline{P R}$, but not at its endpoints, such that PS and QS are also integers.

5/2/11. In $\triangle A B C, \mathrm{AC}>\mathrm{BC}, \mathrm{CM}$ is the median, and CH is the altitude emanating from C , as shown in the figure on the right. Determine the measure of $\angle M C H$ if $\angle A C M$ and $\angle B C H$ each have measure $17^{\circ}$.

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Complete, well-written solutions to at least two of the problems above, accompanied by a completed Cover Sheet should be sent to the following address and postmarked no later than
November 15, 1999. Each participant is expected to develop solutions without help from others.
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